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JOURNAL

OF THE

ELISHA MITCHELL SCIENTIFIC SOCIETY

(Official Publication of the N. C. Academy of Science)

VOL. XX DEC. 1904 NO. 4

PROCEEDINGS OF THE THIRD ANNUAL MEETING OF THE NORTH CAROLINA ACADEMY OF SCIENCE HELD AT WAKE FOREST, N. C., MAY 13-14, 1904.

FRIDAY AFTERNOON SESSION.

The meeting was called to order in Lea Laboratory at 3 P. M. by the President, Dr. Chas. Baskerville, who at once appointed the following Nominating Committee: Messrs. Tait Butler, W. C. Coker and W. L. Poteat.

The Executive Committee had previously elected Dr. F. L. Stevens Vice-President for the present meeting to fill the vacancy caused by the removal from the State of Dr. J. I. Hamaker.

Dr. Baskerville explained delay in the distribution of the last issue of THE JOURNAL containing proceedings of the previous meeting, after which the Academy proceeded at once to the presentation of the following papers:

1. FERTILIZATION OF ALBUGO IPOMOEA PANDURATA: *F. L. Stevens.*

This fungous is multinucleate. The nuclei pass through two mitotic divisions as in other species of the genus. During these divisions all of the nuclei, except one,

[Issued January 28th, 1905.]

pass to the periplasm. One nuclus is retained by the coenocentrum. The fertilization is simple, therefore, and not multiple as in some other species. Minor peculiarities concerning the formation of the walls and development of the coenocentrum, etc., were described.

2. FORECASTS OF THE SKY, OR, EXPECTED NOTABLE OBJECTS: *Jno. F. Lanneau.*

3. A LIRIODENDRON FROM THE DEEP RIVER TRIASSIC: *Collier Cobb.*

The tulip tree is the lone survivor of an ancient race known to extend back into Cretaceous time, from which time its family history has been traced, and "we find this history epitomised in the existing species". Holm considers the primitive ancestral type of Liriodendron to have been a simple, magnolia-like leaf; for not only do all modern relatives of Liridodendron have such leaves, but "there is a progressive simplification and reduction in lobation as we proceed back in time, the most primitive forms known having ovate or oblong simple leaves". The youngest leaves on our modern tulip trees are entire or merely slightly notched, while the mature ones are typically lobed leaves. Hence it is imagined that the primitive tulip tree that grew in early Cretaceous or Jura-Cretaceous time had simple ovate or lanceolate leaves, short petioled and without stipules or bud scales, and with venation much like the existing magnolia leaf. The leaf here presented has the lobate form common in the American Cretaceous, when the modern form is supposed to have become practically fixed in *L. oblongifolium* of the Amboy clays. But this specimen came from the light brown shales of the Deep River Triassic at Cumnock (Egypt), and is found in association with *Macrotaeniopteris magnifolia*, somewhat common in the Triassic coal measures of Virginia, and *Taeniopteris Newberriana*, a well-marked Permian plant. The chief interest in the finding of this plant in the Deep River beds, probably near the base of the Triassic, lies in the fact that it is the mature form of leaf, and

that we must look still farther back in geological time for the simple ovate form.

4. ANGIOSPERMS WITH EXPOSED OVULES: *W. C. Coker.*

Attention was called to some Angiosperms in which the carpels are not completely fused, so that the seeds are exposed directly to the air. In Tiarella the two carpels, which are of unequal size, are not fused even in the flower and the air has full access to the ovules at the time of fertilization. Fruits of Sterculia platanifolia were shown with open carpels and the unripe seeds completely exposed. In Cuphea the central placenta bursts and ruptures the wall of the capsule, exposing the seeds sometime before they are ripe.

5. WORKING UP THE ENTOMOLOGICAL FAUNA OF NORTH CAROLINA: *Franklin Sherman, Jr., and C. S. Brimley.*

(Appears in full in this issue.)

6. SOLUTION OF A NEW MATHEMATICAL CURVE: *Jas. L. Lake.*

7. CLEISTOGAMIC PLANTS: *W. C. Coker.*

The frequent occurrence of cleistogamy was pointed out, and the results of a study of the cleistogamic flowers of Specularia perfoliata and Lamium amplexicaule were presented.

8. REPORT,—RECENT SOIL WORK: *W. G. Morrison.*

The following were presented by title only, the author being absent:

9. FURTHER NOTES ON THE REPRODUCTION OF REPTILES: *C. S. Brimley.*

(Appears in full in this issue.)

10. A CASE OF SNAKE-BITE, BY THE VICTIM: *C. S. Brimley.*

(Appears in full in this issue.)

11. NOTES ON REARING MOTHS FROM THE LARVAE:
C. S. Brimley.

The experience of several years shows that caterpillars of moths, such as the Grape Sphinx (Ampelophaga myron) and the Ash Sphinx (Ceratomia undulosa), whose food is not succulent, may be more easily reared in closed jars than those which feed on succulent leaves, such as Tomato Worms (Phlegethontius Carolina). Hawk moths are often attacked and killed both by Braconid flies and Tachina flies. The large Sphinx of the grape (Pholus pandorus) is even more persecuted by these parasites, and the Tomato Worm is much troubled by the Braconids. The common Ash Sphinx (Ceratomia undulosa) and Grape Sphinx (Ampelophaga myron), however, are seldom molested. Five larvae of the Myron Sphinx of the grape obtained in August all transformed into pupae on consecutive days; two of these produced moths in about sixteen days after transforming; the other three overwintered; two of the latter died, but the third produced a moth almost exactly nine months after it had pupated—taking about fourteen times as long to emerge as the first two. Tomato Worms and some other insects show similar peculiarities in pupation.

NIGHT SESSION.

The night session of the Academy was held in Leigh Hall. The address of welcome was made by Prof. Jno. F. Lanneau and the response by Prof. Collier Cobb. Dr. Baskerville, President of the Academy, then gave his retiring address, the subject being: "*Science and the People.*" At the conclusion of the address the Academy was tendered an informal reception by the ladies and faculty of Wake Forest College.

SATURDAY MORNING SESSION.

The Academy was called to order by President Baskerville and proceeded at once to the presentation of papers, as follows:

12. THE ELECTRONIC THEORY (by title): *C. W. Edwards.*

13. THE PHENOMENA OF FEVER: *Fred. K. Cooke.*

14. ON SEEING THE RED CORPUSCLES IN ONE'S OWN BLOOD: *W. L. Poteat.*

The paper gave in outline a research upon the numerous vaguely bright little discs which pass quickly across the field of vision in all directions when the eye is turned toward a uniformally illuminated surface, as the sky. These bodies are clearly distinguishable from the *muscæ volitantes* and were shown to be the red corpuscles floating in the capillary vessels of the outer layers of the retina in the region of the *macula lutea*. Certain streams of the little bodies were definitely located in the field of vision for each eye. Then the distribution of the blood vessels in the macular region was mapped. In position in the field of vision, as well as in their relation to one another, the vessels and the streams of discs coincided precisely. [So much had been previously observed, first apparently by Vierordt as early as 1856 and afterwards by Professor Rood, Johannes Müller, Helmholtz, and others.] It remained to discover some means of superimposing the two systems upon one another, or of so projecting them as to see streaming discs and branching vessels at the same time. This was done successfully in this way; Through a pin-hole in a card revolved eccentrically a certain vessel projected against the flat flame of an ordinary lamp was localized as passing through the central prominence of the flame. Then with the eye stationary the pin-hole was slowly moved so that the designated vessel could be seen just past the edge of the hole; whereupon the discs (red corpuscles) were plainly visible moving within the walls of the vessel.

15. INACTIVE THORIUM: *Fritz Zerban.*

The paper gives first a brief account of the discovery of radio active substances, of their properties and of the methods for determining and measuring radio activity. Special attention is given to thorium, the

radio activity of which was discovered by G. C. Schmidt and Mme. Curie. The question, whether thorium is primarily radio active or whether its activity is due to the admixture of other radio active substances, is treated. The work carried out by Rutherford and his co-laborers on this subject is briefly stated. While these investigations were made with commercial thorium without regard to its source, Hofmann and Zerban paid strict attention to the latter. From observations made with thorium from all the important thorium minerals the conclusion is drawn that thorium itself possesses no primary radio activity.

16. THORIUM, CAROLINIUM AND BERZELIUM:

Chas. Baskerville.

This paper presents a brief historical account of the discovery of thorium and the questions raised as to its elementary character. The published evidence is considered in conjunction with experimental data obtained, and the conclusions arrived at that thorium is not a primary radio active body. The complex nature of thorium is proved by the conduct of salts with certain organic bases, as phenylhydrazine, for example. Fractions were added giving atomic weights from 212 to 252, the original being 232.5. Pure thorium oxide from several sources was converted into the chloride by heating it, mixed with pure sugar carbon within quart tubes, during the passage of dry chlorine. A volatile chloride, "weisser dampff" of Berzelius, was obtained, decreasing in amount according to the duration and temperature of the reaction. The purified, delicately green oxide obtained from this gave a specific gravity of 8.47 and the element berezelium an atomic weight of 212 (tetrad). The temperature of the tube was raised and thorium tetrachloride distilled away. The residue in the carbon boat on purification gave a greyish pink oxide with a specific gravity of 11.26 and an atomic weight of (tetrad) 255.6 (carolinium). The new thorium, or that in a large measure freed from the berezelium and carolinium, gave an atomic weight of 220.6, and a white oxide with a specific

gravity of 9.2. The original thorium gave atomic weights 232.5 to 232.6, and its oxide had a specific gravity of 10.5. The original thorium dioxide phosphoresces under the influence of ultra violet light, as does zirconium dioxide. Berzelium and carolinium oxides do not respond to this stimulus, while the new thorium glows with increasing luminosity according to the decrease of the novel substances. All these bodies are radio active. Certain chemical differences are noted, as, for example, the conduct of their salts with organic bases, fumaric acid, &c. Carolinium oxide is soluble in concentrated hydrochlroric acid, while berzelium and carolinium oxides are not. Spectral data are wanting; in fact, the limited portions of the spectra (arc and spark) mapped show the bodies identical. The materials are not yet sufficiently pure, nor the spectral data sufficiently complete, to warrant final acceptance, although the preponderance of evidence is favorable to the assumption of the existence of two new members of the family of chemical elements. *

BUSINESS MEETING.

The Academy was called to order at 12:30 P. M. by President Baskerville.

Prof. Poteat urged a continuance of a joint meeting of the Academy with the Chemical Section, and presented the following resolution:

"That the Academy reaffirm its desire that the scientific societies and clubs of the State should hold each one session a year in connection with the annual session of the Academy." *Carried.*

Secretary reported the election to membership of Messrs William B. Streeter and W. C. A. Hammel, of Greensboro.

On recommendation of the Nominating Committee the following officers were elected for the ensuing year:

President, F. L. STEVENS, Raleigh.

Vice-President, JNO. F. LANNEAU, Wake Forest.

Secretary-Treasurer, FRANKLIN SHERMAN, JR., Raleigh.

Executive Committee:—President and Secretary ex officio, CHAS. BASKERVILLE, Chapel Hill; C. W. EDWARDS, Durham; T. GILBERT PEARSON, Greensboro.

On motion it was resolved:

That the North Carolina Academy of Science and the North Carolina Section of the American Chemical Society do hereby express their pleasure and gratitude to the faculty and ladies of Wake Forest College for the facilities placed at their disposal and for the warm hospitality which they have shown at this, their first joint gathering.

Secretary's report was then presented showing a total membership of forty-seven, embracing practically every branch of science represented in the State.

Treasurer's report sdowed a balance of $108.55 on hand.

Prof. Pearson extended an invitation to the Academy to hold the next annual meeting at Greensboro. By motion it was resolved to be the sense of the Academy that the next meeting should be at Greensboro. During the meeting twenty-four persons were in attendance.

The Academy then adjourned.

LIST OF MEMBERS AND ASSOCIATE MEMBERS.

Andrews, W. J., Raleigh.
Ashe, W. W., Raleigh.
Baskerville, Chas., Chapel Hill (removed).
Battle, K. P., Raleigh.
Beardslee, Henry C., Asheville.
Binford, Raymond, Guilford College.
Brewer, Chas. E., Wake Forest.
Brimley, C. S., Raleigh.
Brimley, H. H., Raleigh.
Butler, Tait, Raleigh.
Burkett, Chas., Raleigh.
Cain, Wm., Chapel Hill.
Chapin, Spencer, Littleton.
Cobb, Collier, Chapel Hill.
Coker, R. E., Beaufort.
Coker, W. C., Chapel Hill.

Cooke, Fred. K., Wake Forest.
Duerden, J. E., Chapel Hill (removed).
Edwards, C. W., Durham.
Garrett, Mrs. R. U., Asheville.
Gore, J. W., Chapel Hill.
Hamaker, J. I., Durham (removed).
Hammel, W. C. A., Greensboro.
Hoffman, S. W., Statesville.
Holmes, Jos. A., Chapel Hill.
Howell, E. V., Chapel Hill.
Kesler, J. L., Raleigh (removed).
Kilgore, B. W., Raleigh.
Lake, Jas. L., Wake Forest.
Lanneau, Jno. F., Wake Forest.
Latta, J. E., Chapel Hill.
Lewis, R. H., Raleigh.
Meade, Miss A. M., Raleigh (removed).
Myers, E. W., Greensboro.
Pearson, T. Gilbert, Greensboro.
Pegram, W. H., Durham.
Poteat, W. L., Wake Forest.
Rankin, W. S., Wake Forest.
Sackett, W. G., Raleigh (removed).
Sherman, Franklin, Jr., Raleigh.
Smith, Henry L., Davidson.
Stevens, F. L., Raleigh.
Stevens, Mrs. F. L., Raleigh.
Streeter, Wm. B., Greensboro.
Venable, F, P., Chapel Hill.
Wheeler, A. S., Chapel Hill.
Williams, C. B , Raleigh.
Wilson, H. V., Chapel Hill.
Wilson, R. N., Guilford College.
Winston, Geo. T., Raleigh.

ON THE GRAPHIC REPRESENTATION OF THE PROJECTION OF TWO TRIADS OF PLANES INTO THE MYSTIC HEXAGRAM.

BY ARCHIBALD HENDERSON, PH.D.

WITH PLATE I.

Cayley* has considered the following question: to find a point such that its polar plane in regard to a given system of three planes is the same as its polar plane in regard to another given system of three planes.

Let us designate for convenience the first three planes as a, b, and c, the second three as f, g, and h: The line ab will denote the line of intersection of the planes a and b, and the point abc will denote the point of intersection of the planes a, b, and c; and so in other cases.

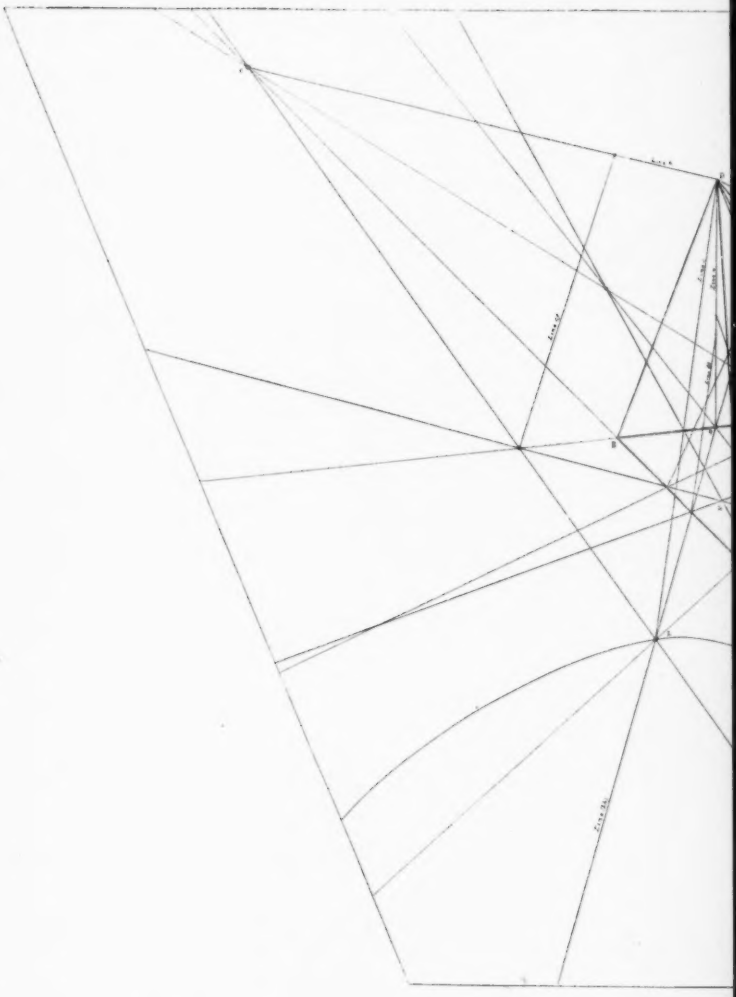
The conclusion reached (l. c.) is that there are four points 0_1, 0_2, 0_3, and 0_4, which fulfil the required conditions. It was shown, that from any one of the points 0, it is possible to draw

a line meeting the lines	$af \times bg \times ch$	[1]
" "	$ag \times bh \times cf$	[2]
" "	$ah \times bf \times cg$	[3]
" "	$af \times bh \times cg$	[4]
" "	$ag \times bf \times ch$	[5]
" "	$ah \times bg \times cf$	[6]

and moreover that these six lines [1], [2], [3], [4], [5], and [6] lie on a cone of the second order. Projecting now the figure of the six planes a, b, c, f, g, and h upon any arbitrary plane (not passing through one of the lines [1], [2], [3], [4], [5], or [6]), we obtain the Pascal configuration. The twenty points abc, abf, ... fgh, are as follows, viz. (omitting the two points abc, fgh) the remaining eighteen points are the

* Collected Math. Papers, Vol. VI., pp. 129-134.

PLATE I.



HENDERSON — GRAPHIC REPRESENTATION.

Pascalian points (the intersections of pairs of lines each through two of the points 1, 2, 3, 4, 5, 6) which lie on the Pascalian lines bc, ca, ab, gh, hf, fg respectively; the point abc is the intersection of the Pascalian lines bc, ca, ab, and the point fgh is the intersection of the Pascalian lines gh, hf, fg, the points in question being two of the points S (Steiner's twenty points, each the intersection of three Pascalian lines).*

§ 2. The question with which I concern myself in the present paper is:—May such a beautiful theorem be represented to the eye in such a way as to show clearly the entire configuration? After some labor and thought, I am able to answer this question in the affirmative. The accompanying plate (Plate I.), drawn to scale with great precision, repre- the sents ultimate result of the investigation.

The initial problem was so to choose the two triads of planes as to give the point of projection 0 a position that might be readily representable on a diagram, showing also the two triads of planes. Choosing to use quadriplanar co-ordinates, I found it desirable for the point 0 to coincide with one of the four vertices A, B, C, and D of the fundamental tetrahedron ABCD. The point 0 was finally chosen, from various considerations into which I shall not enter at present, to coincide with the vertex D. The question that next presented itself was what plane to choose as the plane of projection. The plane ABC was finally chosen as the most economical position for the plane of projection, as will appear later.

I now proceed to an analysis of the problem. The planes were chosen in the following manner:

$$a: \quad w = 0$$
$$b: \quad 256\,x - 384\,y - 96\,z + 459\,w = 0$$
$$c: \quad 96\,x - 64\,y - 256\,z + 153\,w = 0$$

and

$$f: \quad 32\,x + 27\,w = 0$$
$$g: \quad 64\,y - 51\,w = 0$$
$$h: \quad 32\,z - 17\,w = 0$$

*It is to be understood that a point or line and its projection have the same designation, and that the meet of a line such as [1] with the plane of projection is denoted by 1.

The first step is to find the polar planes of a point 0 (x_1, y_1, z_1, w_1) with respect to each one the two triads *abc*, *fgh*, then to identify the equations of these two planes, in order to determine the position of the point 0.

First consider the triad *abc*, given by the equation

$$w\,(256\,x - 384\,y - 96\,z + 459\,w)$$
$$(96\,x - 64\,y - 256\,z + 153\,w) = 0$$

which becomes, on clearing out,

$$\begin{aligned} F\,(x, y, z, w) \equiv\ & 24576\,(x^2 + y^2 + z^2)\,w + 70227\,w^3 - 53248\,xyw \\ & -74752\,xzw + 83232\,xw^2 + 104448\,yzw \\ & -88128\,yw^2 - 132192\,zw^2 = 0 \end{aligned}$$

The polar plane

$$\frac{\delta F}{\delta x_1}x + \frac{\delta F}{\delta y_1}y + \frac{\delta F}{\delta z_1}z + \frac{\delta F}{\delta w_1} = 0$$

of the point (x_1, y_1, z_1, w_1) with respect to this system of three planes is

$$\begin{aligned} & (49152\,x_1 w_1 - 53248\,y_1 w_1 - 74752\,z_1 w_1 + 83232\,w_1^2)\,x \\ & + (49152\,y_1 w_1 - 53248\,x_1 w_1 + 104448\,z_1 w_1 - 88128\,w_1^2)\,y \\ & + (49152\,z_1 w_1 - 74752\,x_1 w_1 + 104448\,y_1 w_1 - 132192\,w_1^2)\,z \\ & + [24576\,(x_1^2 + y_1^2 + z_1^2) + 210681\,w_1^2 - 53248\,x_1 y_1 \\ & \quad - 74752\,x_1 z_1 + 166464\,x_1 w_1 + 104448\,y_1 z_1 - 176256\,y_1 w_1 \\ & \quad - 264384\,z_1 w_1]\,w = 0 \end{aligned} \qquad [1]$$

Also the polar plane

$$\frac{\delta f}{\delta x_1}x + \frac{\delta f}{\delta y_1}y + \frac{\delta f}{\delta z_1}z + \frac{\delta f}{\delta w_1}w = 0$$

of the point (x_1, y_1, z_1, w_1) with respect to the system *fgh*, given by the equation

$$(32\,x + 27\,w)\,(64\,y - 51\,w)\,(32\,z - 17\,w) = 0$$

or

$$\begin{aligned} f\,(x, y, z, w) \equiv\ & 65536\,xyz + 55296\,yzw - 52224\,xzw \\ & - 44064\,zw^2 - 34816\,xyw - 29376\,yw^2 \\ & + 27744\,xw^2 + 23409\,w^3 = 0 \end{aligned}$$

is given by the equation

$$\begin{aligned}&(65536\,y_1z_1 - 52224\,z_1w_1 - 34816\,y_1w_1 + 27744\,w_1^2)\,x\\ &+ (65536\,x_1z_1 + 55296\,z_1w_1 - 34816\,x_1w_1 - 29376\,w_1^2)\,y\\ &+ (65536\,x_1y_1 + 55296\,y_1w_1 - 52224\,x_1w_1 - 44064\,w_1^2)\,z\\ &+ [55296\,y_1z_1 - 52224\,x_1z_1 - 88128\,z_1w_1 - 34816\,x_1y_1\\ &\quad - 58752\,y_1w_1 + 55488\,x_1w_1 + 70227\,w_1^2]\,w = 0 \qquad [2]\end{aligned}$$

Now by inspection it is evident that the equations [1] and [2] are identical (aside from the constant factor 3), if

$$(x_1, y_1, z_1, w_1) = (0, 0, 0, 1).$$

Hence one of the four points 0_1, 0_2, 0_3, 0_4 lies at the vertex D of the fundamental tetrahedron ABCD.

I shall next write down the equations of the nine lines *af*, *ag*, *ah*; *bf*, *bg*, *bh*; *cf*, *cg*, *ch*. The first three may be written as follows:

af:	$x = 0, \quad w = 0$
ag:	$y = 0, \quad w = 0$
ah:	$z = 0, \quad w = 0$

The line *bf* may be written

$$bf: \quad \begin{cases} 3x + 4y + z = 0 \\ 32x + 27w \;\; = 0, \end{cases}$$

since the equation of plane *b* takes the form

$$17\,(32x + 27w) - 96\,(3x + 4y + z) = 0.$$

The line *bg* may be written

$$bg: \quad \begin{cases} 8x + 6y - 3z = 0 \\ 64y - 51w \;\; = 0, \end{cases}$$

since the equation of plane *b* takes the form

$$32\,(8x + 6y - 3z) - 9\,(64y - 51w) = 0.$$

The line *bh* may be written

$$bh: \quad \begin{cases} 2x - 3y + 6z = 0 \\ 32z - 17w \;\; = 0, \end{cases}$$

since the equation of plane *b* takes the form

$$128\,(2x - 3y + 6z) - 27\,(32z - 17w) = 0.$$

In like manner, since the plane c may be written in any one of the forms

$$\begin{aligned} 17\,(32\,x + 27\,w) - 64\,(4\,x + 3\,y + 12\,z) &= 0, \\ 32\,(3\,x + 4\,y - 8\,z) - 3\,(64\,y - 51\,w) &= 0, \\ 32\,(3\,x - 2\,y + z) - 9\,(32\,z - 17\,w) &= 0, \end{aligned}$$

the equations of lines cf, cg, and ch are as follows:

$$cf: \quad \begin{cases} 4\,x + 3\,y + 12\,z = 0 \\ 32\,x + 27\,w = 0 \end{cases}$$

$$cg: \quad \begin{cases} 3\,x + 4\,y - 8\,z = 0 \\ 64\,y - 21\,w = 0 \end{cases}$$

$$ch: \quad \begin{cases} 3\,x - 2\,y + z = 0 \\ 32\,z - 17\,w = 0. \end{cases}$$

We have now to determine the equations of the lines 1, 2, 3, 4, 5, 6, each one of which passes through the point (0, 0, 0, 1) and is conditioned as stated in § 1. Considering line 1, its equations must be of the form

$$\begin{aligned} x + \lambda\,y + \mu\,z &= 0 \\ x + \lambda_1\,y + \mu_1\,z &= 0 \end{aligned}$$

and since it meets line af, we have the condition

$$\begin{vmatrix} 1, & 0, & 0, & 0 \\ 0, & 0, & 0, & 1 \\ 1, & \lambda, & \mu, & 0 \\ 1, & \lambda_1, & \mu_1, & 0 \end{vmatrix} = 0$$

giving $\lambda : \lambda_1 = \mu : \mu_1$ and hence line 1 may be written

$$x = 0, \quad \lambda\,y + \mu\,z = 0$$

Since it also meets line bg, we have the condition

$$\begin{vmatrix} 1, & 0, & 0, & 0 \\ 0, & \lambda, & \mu, & 0 \\ 8, & 6, & -3, & 0 \\ 0, & 64, & 0, & -51 \end{vmatrix} = 0$$

giving $\lambda = -2\mu$, whence the equations of line 1 are

$$1: \qquad x = 0, \quad 2\,y - z = 0$$

That this meets the line *ch* is obvious by inspection. Determining, in similar fashion, the equations of the remaining five lines, we obtain

$$\begin{aligned}
&2: \quad y = 0, \quad x + 3z = 0;\\
&3: \quad x = 0, \quad 3x + 4y = 0;\\
&4: \quad x = 0, \quad y - 2z = 0;\\
&5: \quad y = 0, \quad 3x + z = 0;\\
&6: \quad z = 0, \quad 4x + 3y = 0.
\end{aligned}$$

It is worthy of remark that the six lines 1, 2, 3, 4, 5, 6 lie on the quadric cone, of vertex D, whose equation is

$$12\,(x^2 + y^2 + z^2) - 5\,(6yz - 8zx - 5xy) = 0,$$

since the intersections of this cone by the planes $x = 0$, $y = 0$, $z = 0$ respectively, have for their equations

$$\begin{aligned}
&x = 0, \quad (y - 2z)\,(2y - z) = 0;\\
&y = 0, \quad (x + 3z)\,(3x + z) = 0;\\
&z = 0, \quad (3x + 4y)\,(4x + 3y) = 0.
\end{aligned}$$

§ 3. It is apparent that, since the lines *af*, *ag*, *ah* all lie in the plane ABC ($w = 0$), the most convenient location for the plane of projection is the plane of the face ABC of the fundamental tetrahedron.

I next calculated the co-ordinates of the points where the six lines *bf*, *bg*, *bh*, *cf*, *cg*, *ch* meet two faces of the tetrahedron ABCD. In the case of the lines *af*, *ag*, *ah* no calculation has to be made, while for each one of the lines 1, 2, 3, 4, 5, 6, the co-ordinates of only one point have to be calculated. These results are given below in tabular form:

	x	y	z	w	
bf meets BC	0	—1	4	0	$y = -33\ 3$, $z = 133.3$
" ACD	—27	0	81	32	$x = -27\ 2$, $z = 81{\cdot}6$, $w = 32.2$
bg meets AC	3	0	8	0	$x = 27.2$, $z = 72\ 8$
" BCD	0	51	102	64	$y = 20\ 3$, $z = 40\ 6$, $w = 25.6$
bh meets AB	3	2	0	0	$x = 60$, $y = 40$
" BCD	0	34	17	32	$y = 35.5$, $z = 17.7$, $w = 33.4$
cf meets BC	0	4	—1	0	$y = 133.3$, $z = -33.3$
" ABD	—27	36	0	32	$x = -57$, $y = 76$, $w = 67.6$
cg meets AC	8	0	3	0	$x = 72.8$, $z = 27.2$
" BCD	0	102	51	128	$y = 31.4$, $z = 15.7$, $w = 39\ 5$
ch meets AB	2	3	0	0	$x = 40$, $y = 60$
" BCD	0	17	34	64	$y = 12.8$, $z = 25.6$, $w = 48.2$
1 meets BC	0	1	2	0	$y = 33\ 3$, $z = 66.7$
2 " AC	3	0	—1	0	$x = 150$, $z = -50$
3 " AB	4	—3	0	0	$x = 400$, $y = -300$
4 " BC	0	2	1	0	$y = 66.7$, $z = 33.3$
5 " AC	—1	0	3	0	$x = -50$, $z = 150$
6 " AB	—3	4	0	0	$x = -300$, $y = 400$

The right hand column of co-ordinates was made out for convenience in the construction, the edge of the tetrahedron ABCD (supposed regular) being taken equal to 100 units. Whenever a point lies on an edge of the tetrahedron, the sum of its co-ordinates is equal to 100, and when it lies in one of the faces of the tetrahedron, the sum of its co-ordinates is equal to 86.6, the altitude of the equilateral triangle forming the face.

The argument leading to the conclusion that the projection of the two triads of planes a, b, c; f, g, h from the point D upon the plane ABC ($w = 0$) gives the Pascalian configuration proceeds as follows. Consider the six planes a, b, c, f, g, h and the point of projection D, and the plane of projection ABC—then the line of intersection ab of the planes a and b

will be projected into a line *ab*, and the point of intersection of the planes *a*, *b*, *c* into a point *abc;* and so in the other cases (recalling the fact that the lines *af*, *ag*, *ah* already lie in the plane of projection as chosen). We have thus a plane figure, consisting of the fifteen lines *ab*, *ac*, ... *gh*, and of the twenty points *abc*, *abf*, ... *fgh*; and which is such, that on each of the lines there lie four of the points, and through each of the points there pass three of the lines, viz. the points *abc*, *abf*, *abg*, *abh* lie on the line *ab*; and the lines *bc*, *ca*, *ab* meet in the point *abc*, and so in the other cases.

Moreover, from an inspection of the scheme in § 1, we see that the projections of the lines *af*, *bg*, *ch* meet in a point, and the like for each of the six triads of lines; that is, in the plane figure, we have six points 1, 2, 3, 4, 5, 6, each of them the intersection of three lines as shown in the diagram

$$\begin{aligned} 1 &\equiv af \times bg \times ch \\ 2 &\equiv ag \times bh \times cf \\ 3 &\equiv ah \times bf \times cg \\ 4 &\equiv af \times bh \times cg \\ 5 &\equiv ag \times bf \times ch \\ 6 &\equiv ah \times bg \times cf \end{aligned}$$

and these six points lie in a conic (the intersection of the quadric cone by the plane of projection). It is clear that the lines *af*, *ag*, *ah*; *bf*, *bg*, *bh*; *cf*, *cg*, *ch* are the lines 14, 25, 36; 35, 16, 24; 26, 34, 15 respectively.

Conversely, starting from the points 1, 2, 3, 4, 5, 6 on a conic, and denoting the lines 14, 25, 36; 35, 16, 24; 26, 34, 15 (being, it may be noticed, the sides and diagonals of the hexagon 162435) in the manner just referred to, then it is possible to complete the figure of the fifteen lines *ab*, *ac*, ... *gh* and of the twenty points *abc*, *abf*, ... *fgh*, such that each line contains upon it four points, and that through each point there pass three lines, in the manner already mentioned.

Of the fifteen lines, nine, viz. the lines *af*, *ag*, *ah*; *bf*, *bg*, *bh*; *cf*, *cg*, *ch* are, as has been seen, lines through two of the

six points 1, 2, 3, 4, 5, 6; the remaining lines are *bc*, *ca*, *ab*; *gh*, *hf*, *fg*, These are Pascalian lines

bc	of the	hexagon	162435,
ca	"	"	152634,
ab	"	"	142536,
gh	"	"	152436,
hf	"	"	142635,
fg	"	"	162534,

which appears thus, viz.

line *bc* contains points *bcf*, *bcg*, *bch*.

$= bf \,.\, cf, \quad bg \,.\, cg, \quad bh \,.\, ch$

$= 35 \,.\, 26, \quad 16 \,.\, 34, \quad 24 \,.\, 15;$

that is, *bc* is the Pascalian line of the hexagon 162435; and the like for the rest of the six lines.

The final conclusion has already been stated above in § 1. The drawing explains everything; a few words of explanation and interpretation, however, are perhaps not amiss. By means of four separate scales and the employment of a number of principles of Graphics, I first laid down the tetrahedron ABCD and the fifteen lines in space, *af*, *ag*, *ah*; *bf*, *bg*, *bh*, *cf*, *cg*, *ch*; and 1, 2, 3, 4, 5, 6. In the drawing, the projection of the line *af* is written (*af*), and so in other cases. The projection of the line *bf*, for example, was found by joining the meets of the lines 5 and *bf* with the plane of projection, and similarly for other cases. Only three of the Pascalian lines, viz. *bc*, *ca*, *ab* were drawn, to avoid giving the figure a too complex appearance. The projection of line *ab*, for example, was obtained in the following manner: lines *ag*, *bg* lying in planes *a* and *b* respectively intersect in a point P, say, on the line *ab*; the projections of the points P and P_1 are the meets of the projections of the pairs of lines *ag*, *bg*; *ah*, *bh* respectively. The projection of line *ab* then is the join of the projections of the points P and P_1.

The Steiner point S (shown in the figure), the common meet of the three Pascalian lines *bc*, *ca*, *ab*, is one of the *two*

Steiener points yielded by the projection, the other not being shown for the reason mentioned above.

A final consideration was the construction of the conic section, given by the intersection of the quadric cone (containing the lines 1, 2, 3, 4, 5, 6) with the plane of projection. The six points 1, 2, 3, 4, 5, 6 lie on this conic section and hence it was construeted by points by means of the very theorem under consideration, viz. Pascal's Theorem.*

* To prevent unnecessary complexity on the drawing, only one branch of the hyperbola (the conic, in the present instance) is shown.

WORKING UP THE ENTOMOLOGICAL FAUNA OF NORTH CAROLINA.

FRANKLIN SHERMAN, JR., AND C. S. BRIMLEY.

The authors having joined in an effort to work up as completely as possible the insect life of the State, it is deemed advisable to present a brief account of the work thus far accomplished.

Owing to the vast number of species, the extreme difficulty of classification, and the average comparative small size of the subjects, the task of working up the entire entomological fauna of a State is an unending one—we can only hope to make a start, lay the foundations, and erect as much of the final structure as possible. We can never hope to complete the undertaking. By devoting ourselves more or less closely to certain groups we have been able to get the list of species within those groups fairly well represented in our collections and records. When a species is determined positively to occur within the State the name is written on a card and placed in our permanent files, together with a brief note as to the locality or localities where it is known to occur. To keep this record complete, therefore, we must not only record our own captures but must keep a close watch on the literature and record all authentic observations of others who may at one time or another make collections within our limits.

During 1903 we gave special attention to the Odonata (Dragon-flies) and the Rhopalocera (Butter-flies). Of the Odonata twenty-two species were added to the sixty already known to occur in the State, the most noteworthy addition being probably *Telagrion daecki*, Calvert, only described for the first time in February, 1903, and which was found by Mr. Brimley to be locally abundant near Raleigh in June. The

total number of species of the butterflies was raised to one hundred and three, (this does not include, of course, the night-flying moths), the most interesting additions being *Melitea phaeton*, Drury, bred from larvae taken at Tryon, Polk county, by Mr. W. F. Fiske; *Pamphila carolina*, *Skinner*, taken commonly at Raleigh from March 24th to April 18th by Brimley and Sherman; *Calephelis caenius*, Linn, taken at Fayetteville, July 10th, by Mr. S. W. Foster; *Apatura celtis*, Boisduval and Leconte, taken at Smithfield by Mr. Foster; and three not yet identified Hesperids taken by Mr. Sherman at Washington, Beaufort county, and Beaufort, in Carteret county.

Some work was also done on the first three families of moths, the Sphingidae, Saturniidae and Ceratocampidae, the last two families, which include nearly all of our largest species of moths, being now represented on our records by nearly all the species likely to occur in the State. A list of all the species of these three families now known to occur in North Carolina was published in *Entomological News* for April, 1904, by Mr. Brimley.

In the order Diptera the most exhaustive work was done on the Tabanidae (Horse-flies), forty identified species and one not yet located having been so far secured. The species of this family were identified by Prof. J. S. Hine, of Columbus, Ohio, who has described as new two of the species collected by Mr. Brimley, one of which he has named *Chrysops Brimleyi* in honor of the discoverer. The family Asilidae (Robber-flies) has been explored to some extent, thirty-two species being now known to occur in the State. One of these, *Dasyllus cinerea*, Back, was described as new from specimens taken at Southern Pines in late March, 1903.

In the Coleoptera (Beetles) the list of Cicindelidae (Tiger-beetles) seems about complete. A list of these, comprising nineteen species, was published in *Entomological News* for January, 1904. The Meloidae (Blister-beetles) are now represented in our collections and records by eleven identified and

two not yet identified species, which probably include about all that occur in the State. This order, the Coleoptera, has been more thoroughly explored in a general way than any other of the large orders. In all, one thousand and eight species of Coleoptera are now recorded in our lists from the State.

The order Hemiptera (True Bugs, Scale-insects and Plant-lice) has been gone over in a preliminary way, but not in detail as yet. In this order, the family Coccidae or Scale-insects, which includes such important economic forms as the Scurfy Scale, the Oyster-shell Scale and the San Jose Scale, is now known to be represented by not less than thirty-two species.

Among the Neuropteroid insects (excluding Odonata already mentioned) some work has been done and a few quite rare forms have come to light, among these being *Panorpa rufa*, Gray, previously known by only one specimen from Georgia in an English Museum from which the species had been described. Our specimen was taken at Wilmington on Christmas day, 1902.

The entire list of true insects thus far known to occur in North Carolina as shown by our records includes one thousand seven hundred and ninety-seven species. A number of other species have been collected but not yet identified.

As an incentive to others who may feel an interest in work of this kind it may be stated that there yet remain whole groups of insects in which little or nothing has been done, and many unknown and interesting secrets await the investigation of the patient, industrious, persistent collector.

A CASE OF SNAKE-BITE.

BY THE VICTIM, C. S. BRIMLEY.

On January 21, 1901, I was bitten by a small Cottonmouth (*Ancistrodon piscivorus*) about 260 mm, or 10 2-5 inches long. It happened thus: I had received a number of small snakes tied up in bags to keep them separate, in the pail they were sent in, and there was nothing in the accompanying letter to lead me to think a Cottonmouth was among them (the shipper knew the species). After I had taken this snake out of the bag and seen what it was, I dropped him back again and took hold of it in order to tie it up, when the snake immediately struck at me and drove one fang into the fourth finger of my left hand, on the under side of the last joint.

I sucked the wound well and then ligatured it below the puncture, and immediately went to my doctor (A. O. Jones), taking about one-half an hour to get there. He cupped the wound with a bottle half full of ammonia, getting a good deal of blood, and then injected potassium permanganate into the finger just below the wound which caused it to pain me exceedingly, flooded whole finger with the solution and removed the ligature. I then returned home.

The pain was a sharp smart till about half way to the doctor's, when it nearly ceased. After getting back home finger began to swell and a burning ache set in in wound, which though not very violent kept me awake some time after I had retired. I got to sleep about 12, woke up at 4 A. M. and swelling had spread considerably over back of hand but finger hurt less; a slight feeling of soreness showed itself in my arm both above and below the elbow and in the morning there was a slight soreness at the shoulder. Permanganate bandage was kept on finger till 4 A. M. January 22 and was soaked twice with the solution (after coming home and before

going to bed). Took no alcoholic or other stimulant. Thumb, middle finger and index finger and part of hand and wrist adjacent were not affected. Under side of hand did not swell. My system was not affected at all, except the local trouble in hand and arm.

Swelling on back of hand was all gone by morning of January 24, and by January 26 the swelling and inflammation was mostly confined to the neighborhood of the bite, but there was still some stiffness in the finger and back of hand was still somewhat tender.

The place where the permanganate was injected festered and I pressed pus out of it on January 26. By January 28 my hand was practically well and entirely so by February 1.

FURTHER NOTES ON THE REPRODUCTION OF REPTILES.

BY C. S. BRIMLEY.

On July 18, 1903, five eggs were brought to me, which proved to be those of *Eumeces fasciatus* (Blue-tailed Lizard), these were about the size of Cnemidophorus eggs, measuring 20x12, 20x15, 22x11, the fifth, from which a full grown embryo, ready to hatch and showing the usual markings of the species, had been removed, was not measured. The eggs were very smooth and the skin was very thin, almost transparent, and of a dirty white in color; shape varying from a rather short to a rather long oblong, one of the eggs was one-sided; they were said to have been found near Swift Creek, in this county.

On July 28, 1903, three eggs measuring 30 to 31 mm. long were received. These were long and slender, and were attached to one another on the side, so that their long axes were parallel, one, which was opened, contained a snake embryo too small to identify, and I was not successful in my attempt to hatch them. The probability is that they were the eggs of the Red King Snake (*Ophibolus coccineus.*)

In August, 1903, some *Kinosternon flavescens* in my possession from Austin, Texas, laid two eggs. These were hard-shelled like those of other Mud Turtles, and measured 25x16 and 26x16.

In my former paper read before the Academy in November, 1902, I omitted any mention of *Terrapene carolina* and *T. bauri*, although both species have been hatched out in my "Terrapin pen" from eggs laid in confinement. I have never seen the female in the act of covering her eggs but once, and then she was pressing down the earth as hard as she could. The young of these turtles hatch out in October, unlike those

of our water-turtles, which mainly remain in the shell till the next spring. The eggs which I have dug up in October with young in them ready to hatch are about the size and shape of the eggs of Deirochelys and like them are soft shelled.

I had the pleasure of examining last spring the eggs of the Painted Turtle (*Chrysemys picta*) in situ. The "nest" was situated on a sloping hill-side a little above the reach of the inundations of Walnut Creek, and consisted of a hole in the hard ground in which three eggs had been laid, and the entrance had then been filled with earth and this pressed down hard by the animal's feet. The earth was not in contact with the eggs, which were loose in the cavity below. Externally the nest looked as if some one had thrown a little "pat" of wet clay on the ground and it had dried there. The dirt was so hard it was quite difficult to dig through it to the eggs.

The eggs of Kinosternon on the other hand have the earth packed down hard on them and the entrance to the nest is closed so as to leave a slight depression.

With regard to Lizard eggs Sceloporus lays its eggs in lots of 10 or thereabouts, while the Sand Lizard only lays three at one time.

SOME INTERESTING INSECT CAPTURES.

BY FRANKLIN SHERMAN, JR.

Under this title we wish to record the occurrence in North Carolina of a few of the interesting species of insects which we find from time to time as we are able to study the fauna of the State, and our own collections, more closely.

From the tops of such mountains as Mitchell and Grandfather, to the low swamps of Brunswick county, our State supports a fauna which in richness is equalled by few. One must actually traverse the ground, and study the forms of life in the different regions, to fully realize how great this variation is.

Papilio cresphontes, Cramer, (The Orange Dog). This butterfly, which is doubtless the largest that occurs within our borders, is typically a southern species and is common in Florida, where the caterpillar feeds on the leaves of the orange, thus earning the popular name of The Orange Dog. But in later years the species seems to have been gradually spreading northward and in 1898 the author took one specimen in Maryland, eight miles from Washington. In 1900 three specimens were taken at Ithaca, N. Y., and we have been informed that several others, as well as caterpillars of the species, have since been taken there. It has also recently been taken in Ontario, and Mr. W. J. Holland records it as rather common about Pittsburg. My friend, Mr. C. S. Brimley, who has for years been a thorough student and collector of butterflies in the vicinity of Raleigh, has been unable to locate this species. On August 9th, 1902, one specimen was taken on Shackleford Banks, near Beaufort, Carteret county. It seems likely that in its northern migrations it has remained near the coast, and when the cooler regions are reached, it probably follows the warmer valleys

along the watercourses from which specimens may wander inland. It would still seem strange, however, that the species should remain unknown at Raleigh.

Papilio palamedes, Drury, (Palamedes). This species is even more southern in its distribution than *cresphontes*, and is recorded to occur "from southern Virginia, near the coast, to the extreme southern end of Florida, and westward to southern Missouri and Texas." Like the preceeding, it is not known at Raleigh, and in all probability never occurs there, but two specimens were taken at Beaufort on August 11th, 1902, and at least one or two others were observed at that time.

Thecla damon, Cramer, (The Olive Hairstreak). While this species is probably native to all parts of the State, it is not likely that it is ever abundant, and but one specimen has fallen to me in two years. Mr. Brimley, however, has two or three records of its occurrence at Raleigh. Our one specimen was taken at Durham, on R. F. D. Route No. 2, about 10 miles from town. The little butterfly is exceedingly active, darting quickly in a zig-zag flight which we found it almost impossible to follow with the eye. A peculiar feature which pertains to most of the species of this genus is the very plain unmarked upper surface of the wings, and the usually conspicuously marked under surface. The species can scarcely be distinguished at all by examining the upper surface, but a mere glance at the lower surface decides the matter quickly.

Thecla M-album, Boisduval & Leconte, (The White M-Hairstreak). This pretty little species is said by Holland to have been "taken as far north as New Jersey and Wisconsin, and ranges southward to Venezuela" and that "its citadel is found in the live-oak hummucks of the Gulf States and the oak forests on the highlands of Mexico and more northern countries." This would seem to indicate that in general, the species is too southern in its range to be at all common in North Carolina. Our only specimen, (which is somewhat mutilated), was taken at Blowing Rock on August 29th, 1902. Mr. Brimley does not find the species at Raleigh. Like the

preceding species it seems to be very active in flight.

Thecla halesus, Cramer, (The Great Purple Hairstreak.) This, which we regard as one of the most handsome of all our species, is said by Holland to be "Very common in Central America and Mexico; it is not scarce in the hot parts of the Gulf States, and is even reported as having been captured in southern Illinois. It also occurs in Arizona and southern California." While it is thus assigned as typically a sub-tropical species, we found it not at all rare at Lumberton on September 6th, 1902. Three specimens, representing both sexes, were captured, and one or two others were observed. This is one of the largest species of the genus, and, unlike the others here discussed, it is rather easily taken. One of my specimens was swung at with the net not less than four times, yet made no long flights. It may be that it was only common in the one locality and for a short time, but it seemed to be quite at home. We regard this as a significant capture, indicating that many sub-tropical forms may hereafter be taken in our southeastern section. It is not on record from Raleigh.

Phyciodes nycteis, Doubleday & Hewitson, (Nycteis). The Nycteis butterfly is recorded "from Maine to North Carolina and thence westward to the foothills of the Rockies," and seems to be found only in the western part of our State. Mr. Brimley does not list it from Raleigh, and our one specimen is from Blowing Rock, where it was taken in June, 1901. A somewhat similar species, (*Phyciodes tharos*), is common in all parts of the State.

Grapta faunus, Edwards, (The Faun). Holland says that *faunus* is "found from New England to the Carolinas, thence westward to the Pacific." It is not recorded from Raleigh, and seems to be very local in distribution and only found in the higher colder localities. On August 30th, 1902, it was found not uncommon along the turnpike road from Blowing Rock to Linville. It was my good fortune to be accompanied on that trip by a collector who had had considerable expe-

rience in the Adirondacks, and who stated that it was a common species there. It is not on record from Ithaca, New York, though that region has been very thoroughly collected in.

Pomphopoea unguicularis, Leconte. Our present subject is the largest of the Blister beetles (Meloidae) which we have yet taken in the State. Mr. Schwartz, of the U. S. National Museum, who is probably the best authority on beetles in the country, tells me that it "is sometimes locally common in the mountains." Surely it was plentiful at Blowing Rock on June 26th, 1901. They were feeding on the leaves and fading flowers of the mountain laurel in great numbers. Their scratching among the thick leaves and twigs of the bushes was plainly audible at a distance of a rod or more. Several hundred were taken, and as many more might have been had if desired. Only a few bushes were found to be infested by them, others only a few hundred feet away being unmolested. One party informed us that they had attacked the roses to such an extent that it became necessary to spray them. We also found them abundant on certain peach trees, where they fed on the foliage, and noted the curious fact that they showed a decided preference for leaves that had already been deformed by the Leaf Curl. As many as eight or more were found on a single curled leaf. This fact is also of interest as showing one of the many means by which plant diseases may be dispersed.

MOLECULAR ATTRACTION.*

(THIRD PAPER)

BY J. E. MILLS.

In a preceding paper[1], making use of the measurements of Profs. Ramsay and Young and of Prof. Young, we applied the theoretically derived equation,

$$[1] \qquad \frac{L - E_s}{\sqrt[3]{d} - \sqrt[3]{D}} = \text{constant},$$

to twenty-one substances, and called particular attention to variations in the constants obtained and to the range of temperature covered by the measurements. (In the above equation, L denotes heat of vaporization, E_s is energy spent in overcoming external pressure, and d and D denote density of liquid and vapor respectively.) In this paper making use of the same measurements and the results there derived we wish to point out some further applications of the theory. But first we call attention to several points bearing more directly upon the results of the last paper.

The constant given by equation 1 above, as will appear later in this article, is an important property of a substance, and depends upon the attraction of one molecule for another. We have to refer to this constant so often that a more specific designation is desirable. We have hitherto called the absolute attraction at unit distance from a molecule μ. The above constant we will call μ'. (Therefore $\mu = c_1 \mu' \sqrt[3]{m}$). We call the internal latent heat of vaporization λ and therefore have,

$$[2] \qquad \lambda = \mu' (\sqrt[3]{d} - \sqrt[3]{D}).$$

* Reprinted from the Journal of Physical Chemistry, Vol. 8, No. 9, Dec. p. 593 (1904).

[1] Jour. Phys. Chem., June, 1904. Referred to in this article as "second paper."

THE $\frac{\delta P}{\delta T}$ AT THE CRITICAL TEMPERATURE.

In the second paper we commented upon the difficulty of obtaining correctly the $\frac{\delta P}{\delta T}$ from Biot's formula near the critical temperature. We entirely overlooked the fact that the constant b of the equation, $p = bT - a$, proposed by Profs. Ramsay and Young, was also a $\frac{\delta P}{\delta T}$ and that at the critical temperature (but only at that point, see p. 166) the $\frac{\delta P}{\delta T}$ of the two equations became identical. In work done to establish the truth of the equation, $p = bT - a$, the $\frac{\delta P}{\delta T}$ was obtained for three substances at volumes practically identical with the critical volume. The results are such as to confirm entirely and quantitively the view that the divergences at this point in μ were due to the Biot formula used to obtain the $\frac{\delta P}{\delta T}$.

Thus isopentane gives at the critical temperature (volume 4.266) the $\frac{\delta P}{\delta T} = 367.8$, calculated from Biot's formula. Prof. Young,[1] found from drawn isochors (at the volume 4.3) the value 397, and a calculated value of 407. Therefore Biot's formula gives results about ten per cent. too low, an amount just sufficient to explain the variations in μ' near that point.

For normal pentane Biot's formula at the critical volume, 4.303, gives 364,8 for the $\frac{\delta P}{\delta T}$. Mr. J. Rose-Innes and Dr. Young[2] obtained 407.3 from drawn isochors. Biot's formula therefore shows results too low by about eleven per cent., an amount which is sufficient to explain the low values obtained

[1] Proc. Phys. Soc., 1894-95, p. 650.
[2] Phil. Mag., April, 1899.

in the constant μ' as the critical temperature was approached.

For ethyl oxide Profs. Ramsay and Young[1] obtained at volume 4.00 the $\frac{\delta P}{\delta T} = 413.7$, and it is evident from their paper on ethyl oxide[2] that the values they used for calculating the latent heat of vaporization point to a value at this point of about 490, a fact which sufficiently explains the rise noted in the value of μ'. (The values Profs. Ramsay and Young used for $\frac{\delta P}{\delta T}$ at this point are not in accord with the Biot formula they publish and this prevents quantitive comparison here. See p. 175.)

We have therefore direct proof that the equation, $\frac{L - E_t}{\sqrt[3]{d} - \sqrt[3]{D}} = \text{constant}$, is applicable in the immediate neighborhood of the critical temperature.

THE PRODUCT OF THE PRESSURE AND THE VAPOR DENSITY.

In examining the data used to discover if possible the immediate source of variations in μ' it proved impracticable to plot either the pressure or the volume of the vapor directly against the temperature. But their product, PV, varied more slowly and the values were plotted and gave regular curves except for di-isobutyl, brom-benzene, iodo-benzene, hexamethylene, and water. This result is so interesting that the curves are given below, Diagrams 1 to 3. The numerical results are given in Tables 1 to 21. For water the break in the regularity of the curve occurs at 100° C, and since different formulas were used for the vapor pressure above and below that temperature it would seem certain, in view of remarks already quoted (second paper, p. 396), that the formulas for the pressure need adjustment. For di-isobutyl,

[1] Phil. Mag., May, 1887.
[2] Phil. Trans., 1887A, p. 57.

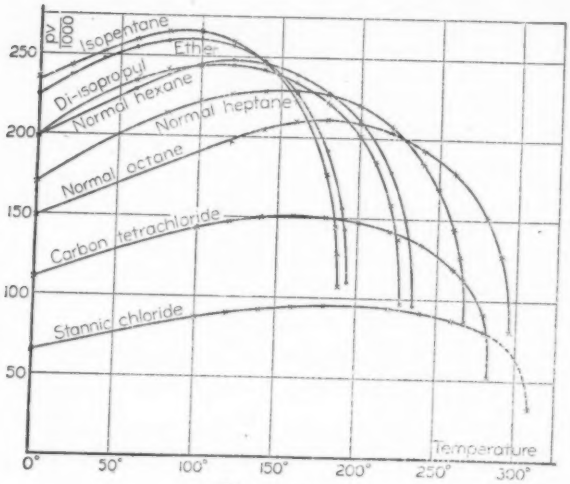


Diagram 1.

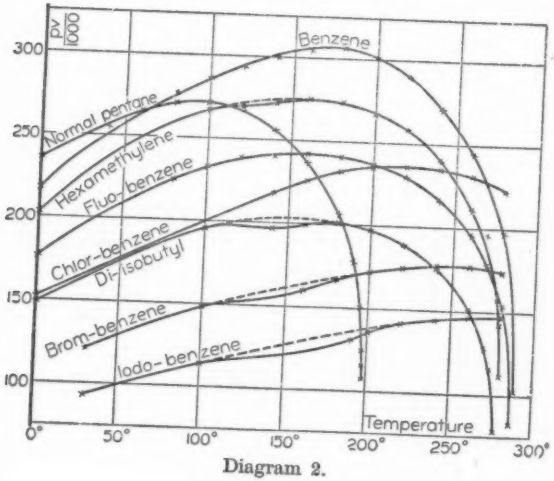


Diagram 2.

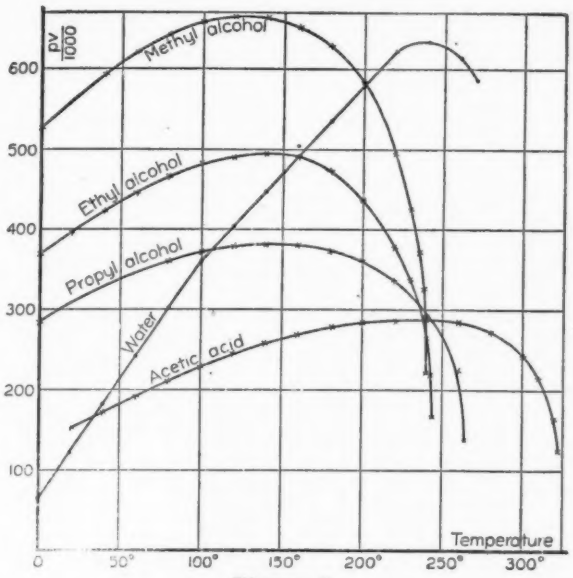


Diagram 3.

For water, add 900 to the ordinates shown by the diagram.

brom-benzene, iodo-benzene, and hexamethylene, the irregularities of the PV curve are more probably due to the vapor density (see second paper, p. 400), and are especially interesting because they occur at just the point where were obtained divergent values for μ'. Reference to the preceding paper will show that di-isobutyl, brom-benzene, and iodo-benzene, were the only substances, except stannic chloride and the associated substances, giving divergences (which have not been explained as due to multiplication of error in calculating the $\frac{\delta P}{\delta T}$) greater than two per cent. from the mean value adopted for μ'. Hexamethylene shows likewise a smaller

though well marked divergence in μ' corresponding to the concave sink in the PV curve.

It is clear therefore that these abnormal values of the PV product must be carefully investigated before the corresponding divergences in μ' can be regarded as in any way evidence against the theory by which that constant is derived.

INTERNAL HEAT OF VAPORIZATION.

The internal heat of vaporization was plotted against the temperature, Diagrams 8 to 10. These curves are more fully discussed later (p. 161). We would here call attention only to one feature of these curves, viz.: Corresponding to divergences in μ' there appear irregularities in these curves and these variations would (excepting in stannic chloride and the associated substances) for the most part disappear if the curves were smoothed, and the smoothed values thus obtained used in the calculations.

DIAGRAM OF $L - E_i$ AGAINST THE $\sqrt[3]{d} - \sqrt[3]{D}$.

We have plotted, Diagrams 4 to 7, the results for all of the

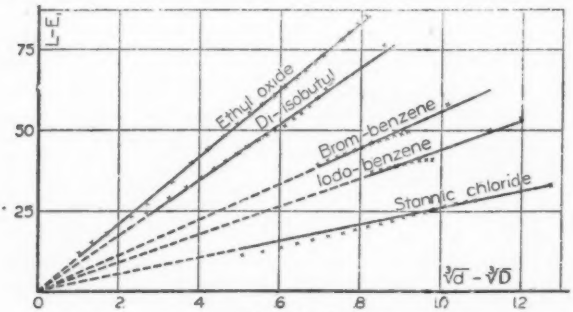


Diagram 4.

Ordinates give internal heat of vaporization in calories. Abscissæ give difference of the cube roots of the densities of liquid and vapor.

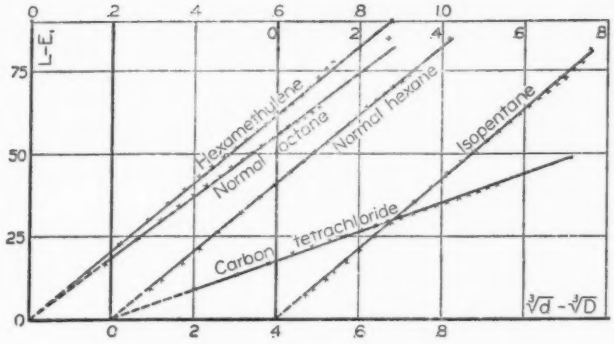


Diagram 5.

Ordinates give internal heat of vaporization in calories. Abscissæ give difference of the cube roots of the densities of liquid and vapor.

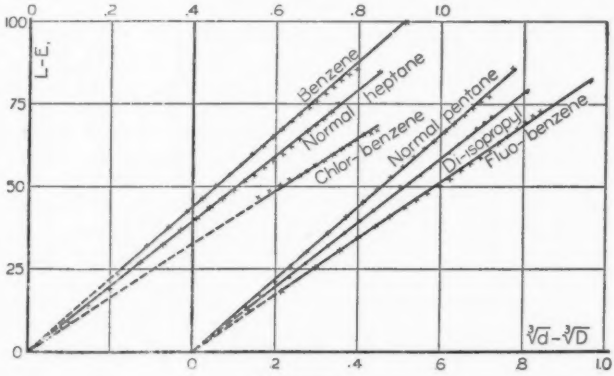


Diagram 6.

Ordinates give internal heat of vaporization in calories. Abscissæ give difference of the cube roots of the densities of liquid and vapor.

substances examined, using the values of $L - E_t$ for ordinates and the values of $\sqrt[3]{d} - \sqrt[3]{D}$ for abscissæ. This is desirable in order to give a more exact idea of the relative extent and importance of the observations. For since neither

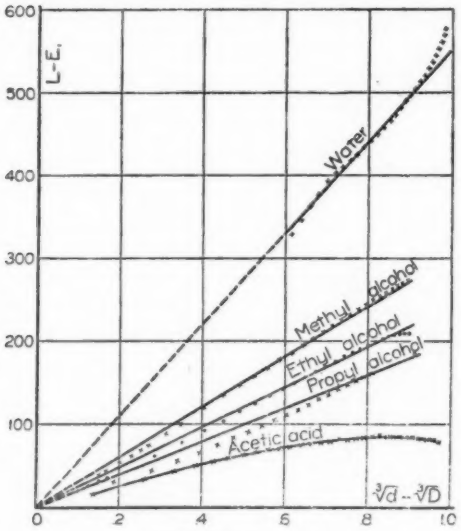


Diagram 7.

Ordinates give internal heat of vaporization in calories. Abscissæ give difference of the cube roots of the densities of liquid and vapor.

$L - E_t$ nor $\sqrt[3]{d} - \sqrt[3]{D}$ vary uniformly with the temperature observations taken every ten degrees are consequently not taken at equal intervals of the curve (a straight line) represented by that equation.

Through each set of observations, excepting for acetic acid, the mean line is drawn. The constants for this mean line were obtained by averaging, for any substance, all of the constants recorded in Table 1 (second paper) that were within two per cent. of the mean values adopted in that table. For di-isobutyl the mean value adopted appeared probably too high and we therefore choose the value 86.3 as being more nearly in accord with the proper constant. The values for these constants are given in Table 25 under the heading μ'.

For ethyl oxide, di-isopropyl, di-isobutyl, isopentane, nor-

mal pentane, normal hexane, normal heptane, normal octane, benzene, hexamethylene, fluo-benzene, carbon tetrachloride, methyl alcohol, ethyl alcohol, propyl alcohol, and acetic acid, sixteen substances in all, the observations are practically complete, extending from near the freezing point of the liquid nearly to the critical temperature. Among these sixteen sub stances the only divergences appearing marked to the eye are normal octane and ethyl alcohol at 0° C, di-isobutyl, those for the alcohols as the lines approach the origin, and acetic acid.

The curves for chlor-benzene, brom-benzene, iodo-benzene, stannic chloride, and water, are not complete. All of these incomplete curves show irregularities and yet it is made most evident by the diagrams, as well as by what has already been said, that with the exception of stannic chloride, the divergences are not so pronounced as to be considered weighty evidence against the theory.

In conclusion therefore we point out that, of the twenty-one substances examined, stannic chloride and the associated substances (methyl alcohol, ethyl alcohol, propyl alcohol, and acetic acid) are the only substances that show variation in μ' without at the same temperature exhibiting irregularities in the data used. That these irregularities in the data are due to the measurements is much to be doubted. But if not so produced they are significant of unknown changes taking place in the substance under examination—changes which were not taken into account in the theory of molecular attraction under discussion and to which that theory as outlined would not be strictly applicable.

THE LATENT HEAT OF VAPORIZATION.

It is interesting to examine more closely and to compare the heats of vaporization calculated by use of the following equations:

[3] $$L = \frac{T}{J}(V - v)\frac{\delta P}{\delta T}.$$

[4] $$L = \mu'(\sqrt[3]{d} - \sqrt[3]{D}) + E_0.$$

[5] $$L = 2RT \log_e \frac{d}{D}.$$

Using the constants we have hitherto adopted, the equations become:

[6] $$L = 0.0_431833\, T (V - v)\frac{\delta P}{\delta T} \text{ cals.}$$

[7] $$L = \mu'(\sqrt[3]{d} - \sqrt[3]{D}) + 0.0_431833\, P (V - v) \text{ cals.}$$

[8] $$L = \frac{9.1522}{m} T \log \frac{d}{D} \text{ cals.},$$

m being the molecular weight, with oxygen equal to 16 as the standard, and logarithms to base ten.

The equations are all theoretically derived.

Equation 3 rests primarily upon the first and second laws of thermodynamics and is deduced therefrom by a familiar line of reasoning. No assumption is made as to the nature or cause of the latent heat, or as to the nature of the substance itself. The equation will serve as well to calculate the latent heat of fusion or the energy absorbed during the change in the crystalline form of a solid. It merely expresses the energy necessary to effect a change in volume under given conditions, and is silent as to the cause of the change or the nature of the substance. So far as present knowledge goes, there is no need for questioning the correctness of the results obtained by this equation, the data being accurate. We can therefore well use the latent heat so obtained as a check, either upon direct measurements of the latent heat, or upon other calculations involving relations and assumptions, which perhaps are true, but which are not so fundamental.

We have already published (second paper, Tables 2 to 22; the calculations for six or seven of the substances were, however, the work of others) the results obtained by the applica-

tion of this equation to twenty-one substances, and only for the sake of comparison repeat a portion of the results in the tables which follow. (See Tables 1 to 21, under the heading Ther.) It must be borne in mind that although these results are accurate where the data is correct, yet errors in the measurements may, and oftentimes certainly do, (because of the method necessarily followed for obtaining the $\frac{\delta P}{\delta T}$) produce far greater proportional errors in the result.

Equation 4 is derived by a simple transposition from equation 2 of the second paper. The assumptions upon which that equation is founded and evidence bearing upon the equation, have been discussed in the preceding paper and here we would only summarize by saying that this equation rests upon the belief that the total kinetic energy of a molecule of a liquid and of its vapor, at the same temperature, are the same; and upon the further assumption that the entire latent heat of vaporization is expended in overcoming the external pressure and in separating the molecules against the action of an attractive force varying inversely as the square of the distance apart of the molecules. The equation is not applicable (*a priori*) if (*a*) the number of molecules change owing to dissociation or decomposition; or if (*b*) the molecules are not evenly distributed throughout the space occupied by them; or if (*c*) for any reason the attraction between these molecules varied with the temperature.

The constant μ' for the twenty-one substances examined are given in Table 25 of this article under the heading μ' and while the constants there given were obtained by a comparison of this equation with the thermodynamical results obtained by use of equation 3, it is easily seen that such a method of obtaining the constant is, we might say, incidental, and was only adopted for the sake of accuracy and convenience. One accurate measurement at *any* temperature of the latent heat of vaporization of any substance to which the equation is applicable, together with a measurement of its

vapor pressure and of the densities of liquid and vapor, would enable the constant for that substance to be calculated. The equation has, therefore, no connection with the thermodynamical equation 3, but rests independently, partly upon the same and partly upon additional assumptions.

The results obtained for the twenty-one substances are given later, Tables 1 to 21, in the columns marked Mills.

In comparing the latent heats so calculated with those obtained from the thermodynamical equation we find that if the five associated subtances be omitted, there are only four determinations in which the results as calculated by the two equations differ by so much as two calories, viz.: di-isobutyl at 0°, normal octane at 0°, chlor-benzene at 270°, and brom-benzene at 30°—in every instance the divergences being at the end point of the Biot curve and thus making it probable that all of these divergences are due to the thermodynamical equation. Excepting stannic chloride, there are only twelve other substances in which the divergence is greater than one calorie. All of these divergences are marked with an asterisk in the tables. This comparison emphasizes more clearly than is possible in any other way the correctness of the law of molecular attraction we have assumed.

Accurate calorimetric measurements are exceedingly difficult even under ordinary pressures and it is not too much to say that where equation 4 is applicable, latent heats calculated with its aid will be more accurate than direct measurements of that quantity, unless very great care is taken in the measurements.

Equations 3 and 4 give with the associated substances and with stannic chloride an agreement which is partial but not complete, divergences being shown at the higher temperatures. One or more of the modifying causes mentioned on page 155 may here be operative.

Equation 5 was deduced by Mr. H. Crompton. Mr. Crompton considers the change in density as if it were due to pressure alone, then in order to keep the substance at that density

without the excess of pressure doubles the amount of energy required. Thus he has,

$$[9] \qquad L = 2\int_{v}^{V} p\delta v = 2RT \log_e \frac{d}{D}.$$

Mr. Crompton makes no supposition as to the true cause of the change of density. But he proceeds on the principle that in effecting a given change of condition the process pursued is immaterial if the total energy change is alone to be considered, the idea being that the change in density could, theoretically, have been produced by pressure. The law governing the change of pressure with the density is known, therefore the amount of energy involved in the change of state can be calculated.

Mr. Crompton uses as the law governing the change of pressure with the volume, the gas equation, PV = RT. But since Mr. Crompton deals with an ideal condition, from which the action of forces other than the pressure are by assumption removed, his equation is not limited to those temperatures for which that equation holds true. It is necessary, however, that PV = RT should represent the true law of pressure for the substance, which is ideally considered only in a limited sense. (The material size of the molecules or some effect of the temperature, etc., *might*, therefore, affect the exactness of the law.) It will be further recognized that Mr. Crompton's equation, no less than equation 4 above, involves the assumption that the only energy change is that involved in a change in density—that is a change of potential energy,—and the total kinetic energy of the molecules of the liquid and of the gas must be the same. As a consequence it is to be expected, though not with certainty owing to possible compensation, that the theory would not apply to substances more associated in the liquid than in the gaseous condition. In the tables below, 1 to 21, in the column headed "Crompton" we give the results obtained from this equation for the substances under examination. For additional evidence bear-

ing upon the theory see Jour. Phys. Chem., April, 1902, p. 219 and Proc. Chem. Soc., Vol. 17, 233, 1901.

It appears that *at low temperatures, where the vapor pressure is small, the results are invariably, and usually very considerably, too large. But at the highest temperatures examined for each substance the agreement is good*, the divergence at this temperature being greater than one calorie only in the cases of ether, normal octane, stannic chloride, and four of the five associated substances.

The results therefore merit detailed study, Of the twenty-one substances, twelve, viz.: ether, di-isopropyl, isopentane, normal pentane, normal hexane, benzene, hexamethylene, fluo-benzene, chlor-benzene, brom-benzene, iodo-benzene, and carbon tetrachloride, give results that are in all respects similar. With them at the lower temperatures Crompton's theory gives too high results, but as the temperature is raised the results grow in the main continually closer to those given by the thermodynamical equation. For these substances after a vapor pressure of 7000 mms. has been reached, it may be said that Crompton's theory gives a very fair approximation, usually within one calorie, to the results obtained thermodynamically.

Di-isobutyl, normal heptane, and normal octane, each show a good agreement at the lowest temperature for which the vapor pressure was measured, viz.; 100°, 80°, and 120°. Then with increasing temperature Crompton's equation gives results lower than those obtained thermodynamically.

Stannic chloride is similar to the above in showing better agreement at the lower temperatures than at the higher, but in this case Crompton's results are uniformly the larger.

In the case of water the results of Crompton are always the larger, but had the observations been continued nearer the critical temperature it is quite possible that good agreement would have been reached.

With the three alcohols, Crompton's equation gives entirely too high results at the lower temperatures. Then with rise

in temperature the results from Crompton's equation become decidedly the lower, but near the critical temperature the difference is not very marked.

With acetic acid, Crompton's equation gives results more than twice too large at the lowest temperature. As the temperature is raised the disagreement becomes continually less. It must be borne in mind that acetic acid vapor shows marked association.

Comparing the three equations, the value of Crompton's theory becomes doubly apparent. Crompton's equation does not involve the vapor pressure and therefore, if trustworthy, will act as a check upon the thermodynamical equation at the end points of the curve, where, owing to the manner of obtaining the $\frac{\delta P}{\delta T}$, the thermodynamical results are somewhat uncertain. On the other hand compared with equation 4, Crompton's theory does not depend upon the attraction and would not be affected by a variation of the attraction with the temperature. It should therefore furnish a clue to those substances in which the molecular attraction does not remain constant with increasing temperature.

At low temperatures Crompton's equation gives values uniformly too high and it therefore cannot be used to check the results of the other equations. But at the highest temperatures the evidence obtained from the results is exceedingly interesting. For ether, di-isopropryl, isopentane, normal pentane, normal hexane, benzene, hexamethylene, fluo-benzene, and carbon tetrachloride, Crompton's equation gives results in better accord with the values obtained from equation 4 than with the thermodynamical results. For these substances at the highest temperatures considered, Crompton's theory gives results differing in no case from those of equation 4 by so much as 0.4 of a calorie, and in several cases the agreement is almost exact. This is splendid confirmation of our belief that in these cases the divergence in the con-

stant of equation, $\frac{L - E_1}{\sqrt[3]{d} - \sqrt[3]{D}} = \text{constant}$, were due to the Biot formula from which the $\frac{\delta P}{\delta T}$ was obtained.

For di-isobutyl, normal heptane, and normal octane, Crompton's equation does not confirm the results of equation 4 when the constants that we have adopted are used, but points instead to lower values for these constants, and we would here call attention to the fact that this indication meets further confirmation (Cf results equations 20, 21 and 25).

For chlor-benzene, brom-benzene, and iodo-benzene, Crompton's theory points to higher constants. This evidence cannot be entirely trusted since the highest measurements for these substances are considerably below the critical temperature.

For stannic chloride, Crompton's equations confirms in a measure the constant adopted. This was a surprise, and suggsets the possibility of an error in the Biot formula used.

For the alcohols and acetic acid, Crompton's equation confirms the belief that in these substances the molecular attraction changes at high temperatures.

To consider the question as to why Crompton's theory does not give correct results at low temperatures we would call attention to the fact that Mr. Crompton could as well have taken the law of the vapor pressure as $PV = P_1V_1$ and have obtained,

[10] $$L = 2 P_1V_1 \log_e \frac{d}{D}.$$

Here V_1 is the volume of the liquid and P_1 is the theoretical pressure of the liquid. In calculating this theoretical pressure it will be seen that the equation cancels back to its original form, $2RT \log_e \frac{d}{D}$. We have here called attention to the transposition only that the equation might be recognized as identically the same equation with which we have to deal in the Joule-Thomson effect of the free expansion of gases. The

Joule-Thomson energy change is in reality a latent heat—the very same effect of the Crompton equation, only the compression is not carried to liquefaction. That effect has been but little studied and is usually laid entirely upon cohesive forces. This may not be the case, and certainly is not the case when hydrogen, which gives negative results, is considered. We here point out that *if it is experimentally possible, a continuation and extension of the experiments of Joule and Lord Kelvin in connection with the theory of Crompton, should enable Crompton's theory to be understood and correctly modified.*

If the PV curves, Diagrams 1 to 3, be examined in connection with Crompton's equation, it will be noted that Crompton's equation usually gives good agreement with the thermodynamical results at points corresponding to the descending portions of those curves.

THE VARIATION OF THE HEAT OF VAPORIZATION WITH THE TEMPERATURE.

The discussion in this and the previous paper of the data bearing upon the latent heats of vaporization obtained for the twenty-one substances examined cannot have failed to impress one with the wonderful accuracy of the measurements by Profs. Ramsay and Young and by Prof. Young, upon which that data is based. The data upon heats of vaporization here made available is therefore the most extensive and the most accurate yet published. The variation of the latent heat with the temperature has always been a question of interest, and we therefore attempt to show most clearly the manner of this variation.

The function of a portion of the latent heat is well known. It is expended in overcoming the external pressure. The portion so expended can be calculated and neither theoretically nor actually does it appear to be a simple function of the temperature. We have called the energy so expended E_i and

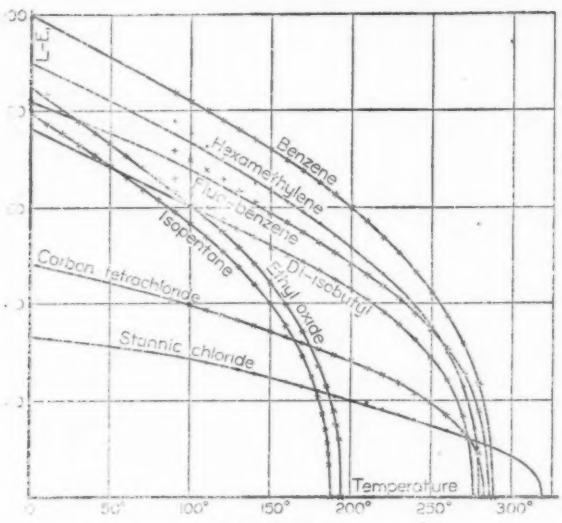


Diagram 8.

Ordinates give internal heat of vaporization in calories.

have given its value for each substance and temperature examined. (See second paper, Tables 2 to 22, under heading E_i).

It has been suggested that $\frac{E_i}{L}$ is a constant, Since E_i is $0.0_431833\ P(V - v)$ and $L = 0.0_431833\ T \frac{\delta P}{\delta T}(V - v)$, this would require that $\frac{E_i}{L} = PT \frac{\delta P}{\delta T} = \text{constant}$. This relation is not confirmed. Examination shows that E_i first increases and then decreases with the temperature, while the latent heat, almost invariably, decreases. E_i thus varies independently of the latent heat and ranges usually between seven and fourteen per cent. of its value.

It was therefore thought best to subtract from the total

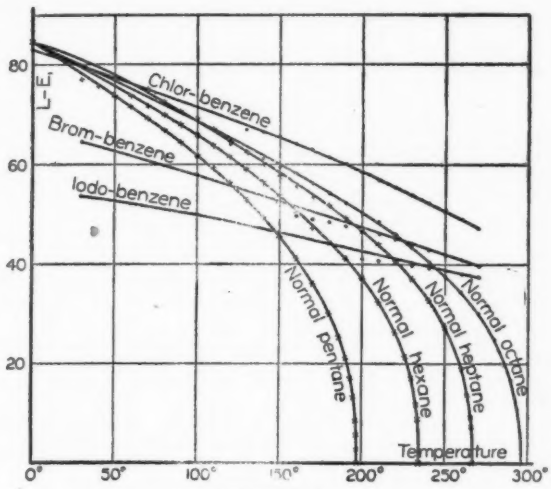


Diagram 9.

Ordinates give internal heat of vaporization in calories.

latent heat this variable amount of energy thus externally expended and to plot the internal latent heat of vaporization against the temperature. The values are given in second paper, Tables 2 to 22, under the heading $L_t - E_t$. The curves are shown in Diagrams, 8, 9, and 10. The observations are marked with dots, or crosses. The scale needs no explanation except that for water 300 should be added to the ordinates to make the reading as expressed in calories correct.

It will be noted:

1. That the internal latent heat cannot be regarded as a linear function of the temperature except at low vapor pressures and for a limited range of temperature. Water does show a linear variation to 230°, but this is due to Regnault's linear formula from which the values were obtained. The values obtained from Ramsay and Young's observations above

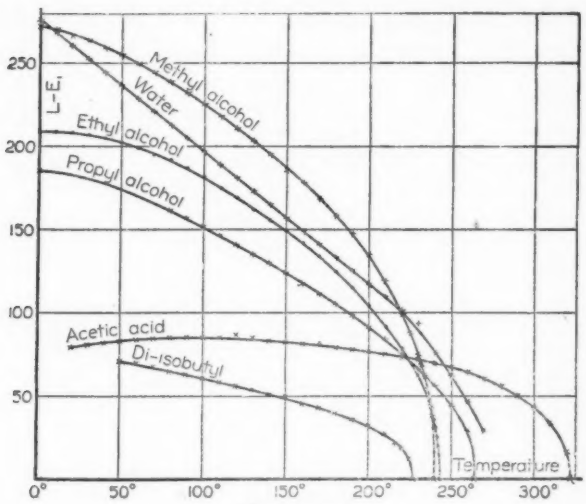


Diagram 10.

Ordinates give internal heat of vaporization in calories.

30° show beyond question that the line for water should also curve perceptibly before that temperature is reached.

2. The curves are all concave towards the temperature axis. Hexamethylene, di-isobutyl, brom-benzene, and iodobenzene would not always be concave if the observations were exactly followed. But the diagrams themselves add strongly to evidence already pointed out (p. 147, etc.) in indicating that these values need further study.

3. Except in the case of acetic acid, the internal latent heat always decreases with the temperature.

4. Several attempts to find a simple empirical formula connecting the internal latent heat and the temperature failed. The investigation was not pushed.

SOME RELATIONS RESULTING FROM THE LATENT HEAT EQUATIONS.

By combining equations 6 and 7 we get:

$$[11] \qquad P = \frac{\delta P}{\delta T} T - \frac{\mu' (\sqrt[3]{d} - \sqrt[3]{D})}{0.0_431833 (V - v)}, \text{ or}$$

$$[12] \qquad P = \frac{\delta P}{\delta T} T - \frac{31414\, \mu'}{V v^{1/3} + V^{2/3} v^{2/3} + V^{1/3} v}.$$

Equations 11 and 12 are not suitable for calculating accurately the pressure, for that value appears as the difference of two comparatively equal values. We give as an example of such application Table 22. Of the substances we have discussed isopentane is one of the most carefully measured and the agreement is to within the limit of experimental error permitted by the equation. The $\frac{\delta P}{\delta T}$ is obtained from Biot's formula. $\frac{31414\, \mu'}{V v^{1/3} + V^{2/3} v^{2/3} + V^{1/3} v}$ is given under the heading "*a*".

Equation 12 will recall the similar equation:

$$[13] \qquad p = bT - a,$$

proposed by Profs. Ramsay and Young[1], where $b = \frac{\delta P}{\delta T}$ and a = constant. The equation is applicable where the volume of the gas is kept constant. "*a*" has a value dependent upon the volume.

The $\frac{\delta P}{\delta T}$ of equation 12 shows the variation of the vapor pressure of a liquid with a rise in temperature, the volume of the liquid meanwhile undergoing change. The $\frac{\delta P}{\delta T}$ of equation 13 denotes a change in pressure of a gas, the volume

[1] Phil. Mag., May, 1887. Phil. Mag., August, 1887. Proc. Phys. Soc., 1894-95. Proc. Phys. Soc., Vol. 15. Phil. Mag., April 1899. Proc. Phys. Soc., Vol. 17.

being kept constant. A liquid cannot exist above its critical temperature, and above that point the liquid and its vapor are identical. The critical temperature therefore marks the limit for which the $\frac{\delta P}{\delta T}$ of equation 12 can be obtained, but just at that point the $\frac{\delta P}{\delta T}$ of the liquid and the $\frac{\delta P}{\delta T}$ of its vapor at constant volume must be identical. That the $\frac{\delta P}{\delta T}$ obtained from the Biot formula could not in the nature of the case be accurate at or near the critical temperature we have already pointed out (second paper, p. 395). That the variation we were there led to expect is quantitively equal to the actual variation as found for equation 13 we have subsequently shown (p. 146). It remains to be seen if "a" of equation 13 corresponds to the function $\frac{31414\,\mu'}{V v^{1/3} + V^{2/3} v^{2/3} + V^{1/3} v}$ of equation 12.

At the critical temperature $v = V$, and therefore we have at that point,

[14] $$a = \frac{10471\,\mu'}{V^{4/3}}.$$

Choosing isopentane as being one of the most carefully measured substances, we found for μ' the value 105.4 (Table 25) and V is 4.266. "a" therefore becomes 159,400. The values given by Dr. Young[1] at volume 4.3 are 157,880 from drawn isochors, and 162,890 when calculated from some values of b. The agreement in this instance is therefore to be regarded as perfect.

That the laws of attraction we have assumed enable the constants of the equation of Ramsay and Young at one point to be foreseen and calculated is proof of the most convincing nature that the theory of the attraction outlined is correct. We are led to believe that, properly modified, the same considerations will elsewhere be successful in a further calculation

[1] Proc. Phys. Soc., 1894-95, p. 654.

of these constants. The relation is so full of possibilities and for its adequate consideration will require so extended an investigation that we postpone the discussion for a separate paper.

Again, combining equations 6 and 8 we obtain:

$$\frac{\delta P}{\delta T} = \frac{287500}{m} \frac{\log \frac{V}{v}}{V - v}. \qquad [15]$$

Owing to the inaccuracy of Crompton's equation at low vapor pressures at such points equation 15 cannot give accurate results. But as the critical temperature is approached we believe that this equation offers the most accurate method yet available for finding the $\frac{\delta P}{\delta T}$, giving results far better than could be obtained from direct observations of the pressure even when the observations are afterwards smoothed.

In Table 23 we apply the equation to isopentane comparing the results obtained by its aid with those obtained from Biot's formula.

At the critical temperature the fraction $\frac{\log \frac{V}{v}}{V - v}$ assumes the indeterminate form $\frac{0}{0}$. Evaluating by diffentiating the numerator and denominator we find the limit approached at the critical temperature to be $\frac{0.4343}{V}$, 0.4343 being the modulus of the Naperian system of logarithms. Therefore we have at the critical temperature,

$$\frac{\delta P}{\delta T} = \frac{124860}{mV}, \qquad [16]$$

a very simple relation. For isopentane we thus get the values of $\frac{\delta P}{\delta T}$ as 405.9, a result in exact accord with the values 397 to 407 as given by Dr. Young[1], and thus confirm-

[1] Proc. Phys. Soc., 1894-95, p. 950.

ing every conclusion we have hitherto drawn relative to the value of $\frac{\delta P}{\delta T}$ at this point.

Combining equation 16 with 12 and remembering that at the critical temperature $V = v$ we have,

$$[17] \qquad P = \frac{124860\,T}{mV} - \frac{10471\,\mu'}{V^{4/3}}.$$

Applying this equation to isopentane, we have, $V = 4.266$, $m = 72.1$, $T = 460.8$, $\mu' = 105.4$, and the pressure thus calculated is 27600 against 25000 observed, an agreement well within the limit of experimental error, since the pressure is found as a difference.

A more general equation is obtained by combining equations 7 and 8 to obtain

$$[18] \qquad P = \frac{31414}{V - v}\left\{\frac{9.1522}{m}T \log \frac{V}{v} - \mu'\left(\sqrt[3]{d} - \sqrt[3]{D}\right)\right\},$$

an equation which, owing to the divergence shown by Crompton's theory is not applicable to low pressures.

We have purposely omitted all reference to those equations connecting the latent heat with the specific heat of liquid or vapor, as it is our purpose at a future time to point out a relation existing between the specific heat of solid, liquid, and vapor, and to discuss such equations in that connection.

A FURTHER APPLICATION OF CROMPTON'S THEORY TO VERIFY THE PROPOSED LAW OF MOLECULAR ATTRACTION.

Following a line of argument already advanced[1], if we consider any gas it is reduced to the liquid state by pressure and by the molecular attraction. In nature the two, pressure and attraction, act jointly and continuously. But theoretically we can separate their action, since mechanically all forces are independent of each other. Legitimately then, we can consider a liquid at its critical temperature as reduced to that

[1] Jour. Phys. Chem., April, 1902, p. 223.

density: First, by the action of a pressure; second, by the action of molecular attraction. Accordingly the theoretical density of the gas at its critical temperature and under its critical pressure was calculated. *The gas would be reduced to that condition if there were no molecular attraction. The remainder of the condensation, to the actual density, must be the work of the attraction alone.*

The theoretical critical density can be calculated by the equation,

$$[19] \qquad D = 0.0_4 16014 \frac{Pm}{T}.$$

If the attraction obeys the law assumed we can use equation 2 to calculate the energy necessary to overcome the attraction and expand the gas from its observed to its theoretical density. If to the energy so calculated we add the energy necessary to overcome the external pressure during the change in volume, we have the total energy, $\lambda + E_i$, required by the change. The equation will become,

$$[20] \quad \lambda + E_i = \mu' (\sqrt[3]{d} - \sqrt[3]{D}) + 0.0_4 31833 P\left(\frac{1}{D} - \frac{1}{d}\right) \text{Cals.}$$

By Crompton's theory we can calculate the energy necessary to change the gas from its observed to its theoretical density as if the change were produced by pressure alone, the equation being:

$$[21] \qquad L = \lambda + E_i = \frac{9.1522}{m} T \log \frac{d}{D} \text{ Cals.}$$

In these equations T is the critical temperature, d denotes the critical density, and D is the theoretical density of the vapor at the critical point.

The results of equation 20 are given in Table 24 under heading Mills. The results from equation 21 are given under the heading Crompton. The difference is also given The agreement is as perfect as could be desired. The difference is usually less than one calorie and amounts to a divergence of more than four per cent. only in the case of normal octane and the associated substances. (To these lat-

ter neither theory is applicable.)

With the three alcohols others have concluded that at the critical temperature *there is no association.* With them the molecular attraction, it will be recalled, changed with the temperature. Using the values of μ', obtained nearest the critical temperature, where the effect of the association could be assumed *nil*, we obtained the last values given, which are in good accord with the results given by Crompton.

In Table 24 the critical data, except for water, are from the measurements of Profs. Ramsay and Young or Dr. Young. References have been given (second paper).

There is nothing in the method adopted to prevent the application of equations 20 and 21 to other points than the critical temperature. Had the equations been combined, an equation similar to equation 18 would have been produced, and the above results may be regarded as but a special application of that equation.

EXTENSION OF THE THEORY TO THE ENERGY RELATIONS AT THE CRITICAL TEMPERATURE.

Solving equation 6 of the first paper[1] we get:

[22] $$\mu = c\frac{L - E_i}{\sqrt[3]{d} - \sqrt[3]{D}}\sqrt[3]{m} = c\mu'\sqrt[3]{m}$$

where c is the same constant for all non-associated substances, and m is the molecular weight. $\frac{L - E_i}{\sqrt[3]{d} - \sqrt[3]{D}}$ we have shown to be a constant for any particular substance, have called this constant μ', and have given the average constants for the substances examined (Table 25). μ represents the absolute attraction at unit distance from a molecule and must be regarded as an exceedingly important and a constant property of the molecule. The values obtained for $\frac{\mu}{c}$ are given in Table 25.

[1] Jour. Phys. Chem., 1902, 4, p. 209.

We would here call attention to the fact that for bodies of similar constitution and closely related chemically the values of μ may be the same, though the observations are not sufficiently extended to permit of any definite conclusion. Thus:

Normal pentane, - - - - - -	457.3
Normal hexane, - - - - - -	454.2
Normal heptane, - - - - - -	458.6
Normal octane, - - - - - -	451.1
But isopentane, - - - - - -	438.6

It is well also to note that for the associated substances, water, the alcohols, and to a less degree for acetic acid where it is less associated, the values for μ are greatly larger than for the other substances examined, and this very large attraction is suggestive in view of the conclusion drawn in the second paper that quite possibly the molecular association of these substances was caused by the molecular attraction.

Resuming a line of argument followed in the first paper (p. 228) we can test our conclusions further. In a gas indefinite expansion takes place as the pressure is decreased. This shows that the attraction between the molecules cannot be great enough to make the paths of the molecules closed curves. In a liquid, while undoubtedly many molecules whose velocity is above the average molecular velocity, are continually flying away from the surface, yet it must certainly be the case that most of the molecules are drawn back by the molecular attraction. There must be for each substance a certain temperature at which the molecular attraction, without outside pressure, is just strong enough to overbalance the kinetic translational energy of the average particle and cause it to return to the liquid or solid substance. At this point, if the attraction varies inversely as the square of of the distance between the molecules, we will have from mechanics,

$$[23] \qquad V'^2 = 2\frac{\mu}{R},$$

where V′ is the molecular velocity and R is the distance apart

of the molecules. μ is the absolute attraction at unit distance.

It is a common text book idea that at the critical temperature the kinetic energy of the molecules of a liquid (gas) under the critical pressure just balances the attraction. The idea rests on the diminution and final disappearance of surface tension at the critical temperature, and the fact that a liquid at its critical temperature may be changed to a gas without the addition of external energy, i. e., by an infinitesimal change in pressure, the heat of vaporization being zero. It must then be at this point that equation 23 will hold good. Putting therefore for the molecular velocity, V', its value at this point $\sqrt{\frac{3RcT}{m}}$ (derived as usual, Rc being 83,250,000) and for R its value $c\sqrt[3]{\frac{m}{d}}$, the constant c being unknown, but equal for all for all substances, we get finally,

$$\mu = c' \frac{T}{m} \sqrt{\frac{m}{d}}. \qquad [24]$$

In this equation T and d denote respectively the critical temperature and density, and c' is the same for all substances. Using the values for the critical constants as given in Table 24 we obtain for $\frac{\mu}{c'}$ the values given in Table 25.

Now if our ideas are correct and the absolute attraction μ, given in equations 22 and 24 are the same, and correctly measured, we have a right to combine these equations and get:

$$\frac{\frac{\mu}{c}}{\frac{\mu}{c'}} = \text{constant} = \frac{L - E_c}{\frac{T}{m\sqrt[3]{\frac{m}{d}}}}, \text{ or,}$$

$$\frac{m\mu'\sqrt[3]{d}}{T} = \text{constant}, \qquad [25]$$

where T must be the critical temperature and d must be the critical density.

If therefore the values of $\frac{\mu}{c}$ given in Table 25 be divided by the values of $\frac{\mu}{c'}$ given in the same table, the results should prove constant for all non-associated substances. The result of this division is given in Table 25.

The mean value for the non-associated substances is 10.76. Since the values for μ' are uncertain by about two per cent., and since the critical data cannot be measured accurately, the close agreement can be regarded as exceedingly satisfactory. A review of the data leads the author to believe that isopentane and normal octane are the only variations that are not due largely to the values adopted for μ'.

Associated substances, *a priori*, could not agree with the equations deduced and they do not. But considering them more particularly, it will be seen that if at the critical temperature the molecular association had vanished (as is said to be the case for the alcohols), equation 24 would hold. If instead of the average value obtained for μ' at the lower temperatures, we use the values for the constant obtained near the critical temperature, equation 22 should also hold, simultaneously with equation 24. For water the data near the critical temperature is lacking, but making use of the proper values for the alcohols, we obtain the results obtained in Table 25. These results would evidently be somewhat better if the observtions of μ' could have been obtained yet nearer the critical point, and we are thus led to regard these associated substances as giving a very remarkable confirmation of the theory.

It should be noted that the constant obtained from equation 25 and given in Table 25 is just one-half of the constant obtained in Trouton's formula, $\frac{mL}{T} = \text{constant}$, where T is the boiling point of a substance. That there is a reason for

this fact we propose later to show in connection with a paper applying the law of attraction to the boiling point.

In conclusion we again point out that we are indebted to Drs. Ramsay and Young and to Dr. Young, for nearly every measurement used in this article. And we would again express our great appreciation of the accuracy of these measurements and acknowledge our obligation to them.

SUMMARY.

1. Several facts bearing upon the results of the last paper are discussed. These facts confirm the law of attraction assumed by making it most clear that $\frac{L - E_1}{\sqrt[3]{d} - \sqrt[3]{D}}$ does equal a constant (designated μ'), for normally constituted substances, and that the equation is applicable with equal exactness in the immediate neighborhood of the critical temperature.

2. An equation, $L = 2RT \log_e \frac{d}{D}$, proposed by Mr. H. Crompton was investigated, and it was found that at low temperatures, where the vapor pressure is small, the results given by the equation are invariably, and usually very considerably, too large. But at higher temperatures the results are correct.

3. Crompton's equation was shown at the critical temperature to give results confirming the law of attraction assumed. (Equations 21 and 22.)

4. It was shown that, $\frac{\delta P}{\delta T} = \frac{287500}{m} \frac{\log \frac{V}{v}}{V - v}$, within certain limits.

5. It was shown that at the critical temperature the following relations hold true:

[*a*] $$P = \frac{\delta P}{\delta T} T - \frac{10471\ \mu'}{V^{\frac{4}{3}}},$$

[b] $$\frac{\delta P}{\delta T} = \frac{124860}{mV},$$

[c] $$\frac{m\mu' \sqrt[3]{d}}{T} = \text{constant},$$

the last being an interesting confirmation of the law of attraction assumed.

6. It was shown that at the critical volume "a" of the equation, $p = bT - a$, becomes identical with the term, $\frac{10471\,\mu'}{V^{\frac{2}{3}}}$.

7. The variation of the latent heat of vaporization with the temperature is discussed.

University of North Carolina,
August, 1904.

ADDENDUM.

Since this paper was written, a letter has been received from Dr. Young, giving the values of the $\frac{\delta P}{\delta T}$ for ethyl oxide. The pressures given in the original paper are correct as are the values of the $\frac{\delta P}{\delta T}$ below 180° C. The corrected values above that temperature are given below, together with the corrected values of the heat vaporization and of μ'.

Temp.	$\frac{\delta P}{\delta T}$ corrected	L	L — E,	μ'
185°	354.9	26.18	22.36	97.7
190	374.1	19.21	16.42	96.4
192	382.2	14.66	12.52	93.5
193	385.8	11.17	9.53	90.7
194	389.8	—	—	—
194.45	391.6	—	—	—

Dr. Young gives 194.45° as probably very nearly the true critical temperature, and 3.814 as the true critical volume of a gram. By interpolation of the results given by Profs. Ramsay and Young (Phil. Mag. May, 1887, p. 441) for the equation $P = bT - a$, the true value for the $\frac{\delta P}{\delta T}$ at this volume appears to be 436. Thus the $\frac{\delta P}{\delta T}$ from Biot's formula at the critical temperature is about ten per cent. too low, an amount which is not quite sufficient to explain the decrease in the value of μ' at the higher temperatures. But above 180° C, owing to hydrolysis of the methyl salicylate used as a heating jacket, the data is somewhat uncertain.

It should be noticed that equation 16 of the present paper gives 441.9 as the value of the $\frac{\delta P}{\delta T}$ at the critical temperature, a result which agrees well with the value 436 given below, thus confirming what has been said as to the accuracy of that equation.

November 1, 1904.

TABLE 1.

Ethyl Oxide.

Temperature.	Heat of Vaporization.			PV/1000.
	Ther.	Mills.	Crompton.	
0	93.27	91.59*	99.42	224
20	87.90	87.94	93.42	237
40	83.18	83.85	87.62	247
60	78.84	79.46	81.95	255
80	73.95	74.49	76.08	259
100	68.35	68.89	69.76	260
120	62.63	62.41	62.79	256
140	55.52	55.05	55.04	247
160	45.99	45.98	45.71	228
180	31.58	32.54	32.19	192
190	20.90	20.58	20.25	156
192	17.10	16.11	15.84	143
193	13.67	12.61*	12.36	132
194				110

TABLE 2.

Di-isopropyl.

Temperature.	Heat of Vaporization.			PV/1000.
	Ther.	Mills.	Crompton.	
0	85.11	85.40	94.24	198
60	76.90	76.42	79.91	234
80	72.86	72.66	75.13	241
100	68.28	68.45	70.15	243
120	63.57	63.85	64.98	244
140	58.58	58.75	59.46	242
160	52.70	52.87	53.23	234
180	45.86	45.90	45.99	221
200	37.15	37.16	37.05	201
210	31.09	31.31	31.16	185
220	22.14	22.79	22 59	159
225	14.57	15.40	15.23	137
227.35				96.8

TABLE 3.

Di-isobutyl.

Temperature.	Heat of Vaporization.			PV/1000.
	Ther.	Mills.	Crompton.	
0	80.99	78.62*	91.35	149
100	69.25	67.52*	69.46	196
120	63.79	64 15	64.99	195
140	59.84	60.70	60.78	197
160	56.30	57.01	56.57	199
180	53.05	53.10	52.28	201
200	48.83	48.51	47.43	197
220	43 80	43.09	41.85	189
240	37 37	36.39	35.08	175
260	27.93	27.08	25.88	151
270	19.54	19.29	18.33	128
274	14.24	14.22	13.46	115
276.8				78.8

TABLE 4.

Isopentane.

Temperature.	Heat of Vaporization.			PV/1000.
	Ther.	Mills.	Crompton.	
0	88.86	87.45*	95.92	246
20	82 91	83.52	89.98	243
40	78.69	79.44	84.46	252
60	74.56	75.00	78.96	260
80	70.00	70.12	73.26	265
100	64.78	64.56	67.09	265
120	58.62	58.19	60.16	259
140	51.07	50.52	52.00	246
160	41.27	40.83	41.84	223
180	24.65	25.05	25.54	177
185	16.47	17.17	17.46	153
187	10.43	11 08	11.25	135
187.4	8.07	8.69	8.81	128
187.8				107

TABLE 5.

Normal Pentane.

Temperature.	Heat of Vaporization.			PV / 1000.
	Ther.	Mills.	Crompton.	
0	93.36	92.38	101.18	236
40	83.63	84.58	89.47	255
60	80.04	80.29	83.98	266
80	75.30	75.39	78.18	271
100	69.87	69.88	71.90	271
120	64.48	63.85	65.34	269
140	56.58	56.31	57.30	256
160	47.42	47.27	47.89	237
180	35.01	35.16	35.43	207
190	24.68	25.38	25.49	178
195	15.66	16.60	16.63	151
197	6.55	7.15	7.14	125
197.15	3 11	3.41	3.41	116
197.2				108

TABLE 6.

Normal Hexane.

Temperature.	Heat of Vaporization.			PV / 1000
	Ther.	Mills.	Crompton	
0	90.98	90.33	100.82	198
60	80.82	81.09	84.86	228
80	77.55	77.43	79 90	238
100	73.65	73 37	74.84	244
120	69.38	68.91	69.67	248
140	63.96	63.75	63.98	247
160	57.63	57.78	57.61	239
180	50.93	51.04	50.59	228
200	42.75	42.78	42.16	211
220	30.37	30.82	30.18	179
230	19.73	20.56	20.04	149
234	10.44	11.26	10.94	123
234.8				95.7

TABLE 7

Normal Heptane.

Temperature.	Heat of Vaporization.			PV / 1000.
	Ther.	Mills.	Crompton.	
0	89.86	89.10	100.25	170
80	79.52	79.06	80.66	214
100	75.96	75.76	76.12	222
120	71.79	72.06	71.47	226
140	67.19	67.94	66.69	228
160	62.74	63.47	61.73	229
180	58.52	58.63	56 63	227
200	53.17	52.98	50.83	220
220	46.46	46 09	43.92	207
240	37.45	37.12	35.11	185
260	21.90	22.33	20.90	144
264	15.84	16.48	15.38	127
266.5	8.50	9.15	8.52	107
266.9				87.0

TABLE 8.

Normal Octane.

Temperature.	Heat of Vaporization.			PV / 1000
	Ther.	Mills	Crompton.	
0	89.46	85.55*	99 98	149
120	70.27	71.55*	71.58	197
140	67.56	68.38	67.52	204
160	64.82	64.96	63.45	210
180	60.99	61.07	59.06	211
200	56.87	56.71	54.38	210
220	52.08	51.72	49.20	204
240	45.97	45 65	43.10	193
260	39.14	38.57	36.15	179
280	28.26	28.13	26.12	152
290	19.10	19.47	17.97	128
296.2				80.5

TABLE 9.

Benzene.

Temperature.	Heat of Vaporization.			PV/1000.
	Ther.	Mills.	Crompton.	
0	107.05	107.25	123.89	218
80	94.40	95.75*	102.49	277
100	91.05	92.07*	97.44	286
120	87 36	88.00	92.32	293
140	83.48	83.72	87.27	300
160	79.20	79.18	82.09	305
180	74.53	74.24	76.69	307
200	69.01	68.45	70.43	301
220	62.32	61.74	63.34	290
240	54.21	53.74	54.96	271
260	43.76	43.57	44.40	244
280	27.43	27.79	28.17	197
288.5				102.5

TABLE 10.

Hexamethylene.

Temperature.	Heat of Vaporization.			PV/1000.
	Ther.	Mills.	Crompton.	
0	96.22	97.13	111.80	208
100	83.71	82.82	87.47	267
120	78.69	78.89	82.58	270
140	73.60	74.56	77.53	272
160	69.68	70.19	72.91	275
180	65.15	65.27	67.25	274
200	59.37	59.60	61.21	267
220	53.56	53.27	54.56	258
240	45.76	45.37	46.31	240
260	35.16	34.84	35.42	211
270	26.72	26.76	27.14	196
277	16.85	17.34	17.53	157
279	11.78	12.34	12.45	142
279.95				111

TABLE 11.

Fluo-benzene.

Temperature.	Heat of Vaporization.			PV/1000.
	Ther.	Mills.	Crompton.	
0	87.39	88.36	102.66	177
80	80.06	79.05	84.62	224
100	77.09	76.06*	80.38	233
120	73.21	72.68	76.02	239
140	68.50	68.91	71.46	240
160	64.37	64.96	66.92	242
180	60.09	60.65	62.17	241
200	55.24	55 76	56.89	236
220	50.28	50.32	51.18	230
240	44.03	43.78	44.34	216
260	35.74	35 30	35.59	196
280	20.95	21.15	21.18	153
286.55				82.6

TABLE 12.

Chlor-benzene.

Temperature.	Heat of Vaporization.			PV/1000.
	Ther.	Mills.	Crompton.	
0	87.72	87.25	107.14	152
140	73.37	74.12	79.06	218
160	71.26	71.56	75.63	226
180	68.77	68.75	72.08	232
200	65.65	65.58	68.34	235
220	62.60	62.18	64.46	236
240	59.65	58.48*	60.30	235
260	56.06	54.24*	55.66	231
270	53.95	51.88*	53.10	
280				223
360.7				81.3

Table 13.

Brom-benzene.

Temperature.	Heat of Vaporization.			PV / 1000.
	Ther.	Mills.	Crompton.	
30	68.68	65.78*	79.45	121
100	62.58	61.77	68.76	148
160	54.69	56.55*	60.39	160
180	53.80	54.80	58.02	167
200	52.27	52.82	55.47	172
220	50.41	50.61	52.80	175
240	48.29	48.24	50.02	176
260	46.30	45.72	47.18	177
270	45.03	44.32	45.62	
280				174
397.0				59.5

Table 14.

Iodo-benzene.

Temperature.	Heat of Vaporization.			PV / 1000.
	Ther.	Mills.	Crompton.	
30	56.70	55.99	68.77	92.8
100	53.68	53.32	60.08	114
200	45.34	47.01*	49.36	135
220	44.94	45.64	47.57	142
240	43.70	44.02	45.54	144
260	42.50	42.25	43.45	146
270	41.85	41.27	42.31	
280				147
448				49.7

Table 15.

Carbon Tetrachloride.

Temperature.	Heat of Vaporization.			PV / 1000.
	Ther.	Mills.	Crompton.	
0	51.87	52.50	60.72	111
100	44.20	44.67	47.62	143
120	42.24	42.64	45.13	147
140	40.26	40.51	42.61	150
160	37.95	38.15	39.96	151
180	35.40	35.53	37.08	150
200	32.61	32.58	33.92	147
220	29.45	29.19	30.31	142
240	25.56	25.15	26.05	134
260	20.07	19.73	20.36	119
280	10.43	10.47	10.75	91.2
283.15				52.3

Table 16.

Stannic Chloride.

Temperature.	Heat of Vaporization.			PV / 1000.
	Ther.	Mills.	Crompton.	
0	35.38	35.18	42.19	65.4
100	31.40	30.95	33.29	86.5
120	30.02	29.81	31.67	89.7
140	28.52	28.56	30.02	92.3
160	26.86	27.19	28.38	94.1
180	24.89	25.69	26.66	94.7
200	22.97	24.08*	24.90	94.8
220	20.89	22.28*	22.99	93.9
240	18.60	20.25*	20.88	91.5
260	16.03	17.89*	18.46	87.1
280	13.08	15.06*	15.52	80.3
318.7				33.6

TABLE 17.

Water.

Temperature.	Heat of Vaporization.			PV/1000.
	Ther.	Mills.	Crompton.	
0	606.5	574.6	738.4	965
20	592.6	571.2	709.8	1022
40	578.6	565.8	682.4	1082
60	564.7	558.4	656.0	1142
80	550.6	549.4	630.6	1202
100	536.5	538.9	606.5	1260
120	522.3	526.4	582.7	1304
140	508.0	512.8	559.4	1347
160	493.6	498.1	536.4	1404
180	479.0	481.3	513.3	1435
200	464.3	463.8	490.7	1478
220	449.4	445.2	467.4	1523
240	423.8	423.5	442.1	1533
260	394.0	399.3	414.2	1514
270	374.5	385.4	398.5	1486
364.3				345

TABLE 18.

Methyl Alcohol.

Temperature.	Heat of Vaporization.			PV/1000.
	Ther.	Mills.	Crompton.	
0	289.2	289.4	324.4	527
20	284.5	283.1	307.2	561
40	277.8	275.8	290.3	593
60	269.4	266.9	273.6	621
80	259.0	256.8	256.9	643
100	246.0	245.1	240.0	658
120	232.0	231.7	222.7	665
140	216.1	216.6	205.0	664
160	198.3	199.4	185.8	652
180	177.2	179.3	164.9	629
200	151.8	154.4	140.0	585
220	112.5	118.5	106.2	494
230	84.5	92.2	81.9	426
236	61.7	68.9	61.0	372
238.5	44.2	51.1	45.0	326
240				220

TABLE 19.

Ethyl Alcohol.

Temperature.	Heat of Vaporization.			PV/1000.
	Ther.	Mills.	Crompton.	
0	220.9	228.4	238.0	389
20	220.6	223.9	224.3	396
40	218.7	218.3	210.8	423
60	213.4	211.4	197.5	446
80	206.4	203.4	184.2	467
100	197.1	194.4	171.2	483
120	184.2	183.6	157.9	490
140	171.1	171.2	144.5	494
160	156.9	157.7	130.7	491
180	139.2	141.7	115.6	474
200	116.6	121.7	97.8	437
220	88.2	94.6	74.8	377
240	40.3	45.8	35.5	265
242.5	22.1	26.5	20.3	219
243.6				167

TABLE 20.

Propyl Alcohol.

Temperature.	Heat of Vaporization.			PV/1000.
	Ther.	Mills.	Crompton.	
0	194.4	190.8	200.9	284
80	173.0	172.4	153.7	360
100	164.0	165.2	142.7	373
120	153.0	157.0	132.0	379
140	142.4	147.9	121.5	381
160	129.0	137.6	110.7	381
180	116.3	125.8	99.9	373
200	102.2	112.7	88.2	362
220	85.3	96.6	74.7	336
240	63.4	75.3	57.5	290
260	33.5	41.4	31.2	224
263.7				137

TABLE 21.

Acetic Acid.

Temperature.	Heat of Vaporization.			PV/1000	Temperature.	Heat of Vaporization.			PV/1000.
	Ther.	Mills	Crompton.			Ther.	Mills.	Crompton.	
20	84.05	-------	184.80	153	200	85.55	-------	115.73	284
40	87.02	-------	176.94	173	220	82.02	-------	106.72	286
60	89.69	-------	169.46	192	240	78.18	-------	97.21	288
80	91.59	-------	162.05	211	260	72.26	-------	86.44	284
100	92.32	-------	154.62	228	280	63.39	-------	73.61	272
120	94.38	-------	147.20	246	300	48.95	-------	56.86	243
140	91.83	-------	139.58	259	310	37.77	-------	44.37	216
160	89.63	-------	131.82	269	320	18.04	-------	22.67	164
180	87.71	-------	123.95	278	321.65	-------	-------	-------	124

TABLE 22.

Isopentane.

Temperature.	$\frac{\delta P}{\delta T}$	$\frac{\delta P}{\delta T}T$	a	Pressure.	
				Calculated.	Biot.
0	11.16	3047	2742	305	258
20	21.04	6165	5639	526	573
40	35.66	11162	10139	1023	1131
60	55.73	18558	16634	1924	2036
80	81.85	28893	25541	3352	3401
100	114.70	42783	37310	5473	5354
120	155.18	60986	52480	8506	8040
140	204.42	84425	71920	12505	11620
160	264.0	114312	96810	17502	16285
180	335.9	152163	132410	19753	22262
187.8	367.8	169480	159420	10060	25005

TABLE 23.

Isopentane.

Temperature.	$\frac{V}{\log \frac{V}{v}}$	V − v	$\frac{\log \frac{V}{v}}{V - v}$	$\frac{\delta P}{\delta T}$ from Equati'n 15	$\frac{\delta P}{\delta T}$ from Biot
0	2.768	915.8	.003022	12.05	11.16
20	2.419	422.4	.005725	22.83	21.04
40	2.126	221.5	.009598	38.27	35.66
60	1.868	126.2	.01480	59.00	55.73
80	1.635	76.1	.02147	85.61	81.85
100	1.417	47.56	.02980	118.82	114.70
120	1.206	30.20	.03998	159.22	155.18
140	.9920	19.00	.05221	208.16	204.42
160	.7612	11.34	.06712	267.6	264.0
180	.4442	5.09	.08728	348.0	335.9
187.4	.1508	1.503	.1003	399.9	366.4
187.8	-------	-------	.1018	405.9	367.8

TABLE 24.

Substance.	Molecular Weight.	μ'	Critical.			Theoretical Density.
			Temperature.	Pressure	Density.	
Ether	74.08	104.4	194.0	27060	.2463	.06873
Di-isopropyl	86.11	98.08	227.35	23330	.2411	.06430
Di-isobutyl	114.14	86.30	276.8	18640	.2366	.06197
Isopentane	72.10	105.4	187.8	25005	.2344	.06266
Normal pentane	72.10	109.9	197.2	25063	.2324	.06157
Normal hexane	86.11	102.85	234.8	22433	.2343	.06093
Normal heptane	100.13	98.75	266.9	20899	.2344	.06057
Normal Octane	114.14	96.00	296.2	18734	.2327	.06015
Benzene	78.05	109.5	288 5	36395	.3550	.08100
Hexamethylene	84.10	103.6	279.95	30234	.2733	.07365
Fluo-benzene	96.09	85.60	286.55	33912	.4107	.09325
Chlor-benzene	112.5	81.19	360.7	33962	.4179	.09654
Brom-benzene	157.0	56.12	397.0	33912	.5695	.12780
Iodo-benzene	203.9	44.40	448.0	33912	.6826	.1536
Carbon tetrachloride	153.8	44.09	283.15	34180	.6536	.1514
Stannic chloride	260.8	26.04	318 7	28080	.8621	.1982
Water	18.02	553.3	364.3	148200	.429	.06707
Methyl alcohol	32.03	305.0	240.0	59759	.2715	.05074
Ethyl alcohol	46.05	241.2	243.6	47700	.2857	.06809
Propyl alcohol	60.06	199.2	263.7	38120	.2778	.06830
Acetic Acid	60.03	180.0	321.65	43480	.3506	.07025
Methyl alcohol		259.4				
Ethyl Alcohol		197.2				
Propyl alcohol		157.0				

TABLE 24—Continued.

Substance.	E_1	$d'_S - D'_S$	λ	Latent Heat		Δ
				Mills.	Crompton.	
Ether	9.03	.2173	22.69	31.72	31.98	—.26
Di-isopropyl	8.46	.2218	21.75	30.21	30.52	—.31
Di-isobutyl	7.07	.2228	19.23	26.30	25.65	.65
Isopentane	9.30	.2194	23.11	32.41	33 51	—1.10
Normal pentane	9.52	.2199	24.17	33.69	34.42	—.73
Normal hexane	8.67	.2230	22.93	31.60	31.57	.03
Normal heptane	7.95	.2239	22.11	30.06	29.00	1.06
Normal octane	7.35	.2233	20.77	28.12	26.82	1.30
Benzene	11.04	.2754	30.14	41.18	42.26	—1.08
Hexamethylene	9.55	.2298	23.80	33.35	34.27	—.92
Fluo-benzene	8.95	.2898	24.81	33.76	34.30	—.54
Clor-benzene	8.62	.2889	23.45	32.07	32.81	—.74
Brom-benzene	6.59	.3258	18.28	24.87	25.41	—.54
Iodo-benzene	5.45	.3450	15.32	20.77	20.97	—.20
Carbon tetrachloride	5.52	.3348	14.76	20.28	21.02	—.74
Stannic chloride	3.47	.3686	9.60	13.07	13.25	—.18
Water	59.29	.3479	192.5	251.8	260.9	—9.1
Methyl alcohol	24.84	.2566	78.26	103.10	96.35	6.75
Ethyl alcohol	16.99	.2502	60.35	77.34	63.94	13.50
Propyl alcohol	13.39	.2437	48.55	61.94	49.81	12.13
Acetic acid	15.75	2925	38.02	53.77	63.27	—9.50
Methyl alcohol			66.55	91.39		—4.96
Ethyl alcohol			49.34	66.33		2.39
Propyl alcohol			38.25	51.64		1.83

TABLE 25.

Substance.	μ'	$\frac{\mu}{c}$	$\frac{\mu}{c'}$	Ratio of $\frac{\mu}{c}$ to $\frac{\mu}{c'}$
Ether	104.4	438.5	42.23	10.38
Di-isopropyl	98.08	433.1	41.23	10.51
Di-isobutyl	86.30	418.6	37.78	11.08
Isopentane	105.4	438.6	43.14	10.17
Normal pentane	109.9	457.3	44.15	10.36
Normal hexane	102.85	454.2	42.24	10.75
Normal heptane	98.75	458.6	40.62	11.29
Normal octane	93.00	451.1	39.54	11.47
Benzene	109.5	468.0	43.415	10.78
Hexamethylene	108.6	453.8	44.38	10.23
Fluo-benzene	85.60	392.3	35.86	10.94
Chlor-benzene	81.19	391.9	36.38	10.77
Brom-benzene	56.12	302.8	27.76	10.91
Iodo-benzene	44.40	261.3	23.64	11.05
Carbon tetrachloride	44.09	296.2	22.32	10.58
Stannic chloride	26.04	166.4	15.23	10.93
Water	553.3	1450.8	123.00	11.79
Methyl alcohol	305.0	968.6	78.56	12.34
Ethyl alcohol	241.2	864.5	61.04	14.16
Propyl alcohol	199.2	780.1	53.64	14.54
Acetic acid	130.0	509.1	55.00	9.26
Methyl alcohol	259.4	823.8	----------	10.49
Ethyl alcohol	197.2	706.8	----------	11.58
Propyl alcohol	157.0	614.8	----------	11.46